Algorithmic Sustainable Design: The Future of Architectural Theory.

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Lecture 4

A. Cellular automata.

B. Sierpinski carpets and sea-shells.

C. Design in hyperspace and connection to the sacred.

Introduction

- Unlike the previous lectures, this lecture gives no practical model for design
- Instead, I examine a union of ideas from computer science, physics, mathematics, and spirituality
- Working from analogy, I try to get into the foundations of architecture

Relate architecture to other disciplines

- I relate the basis of architecture to other disciplines
- In the 20th Century, architecture has been isolated from the technological world and all of its impressive advances
- Sure, architects have applied technology, but they worked from an artistic basis

“Toy” models

- Scientists confronted with a highly complex problem often create a “toy model”
- Captures the essentials in a very simple model, which helps to understand the underlying mechanism
- Then work by analogy to solve the real problem

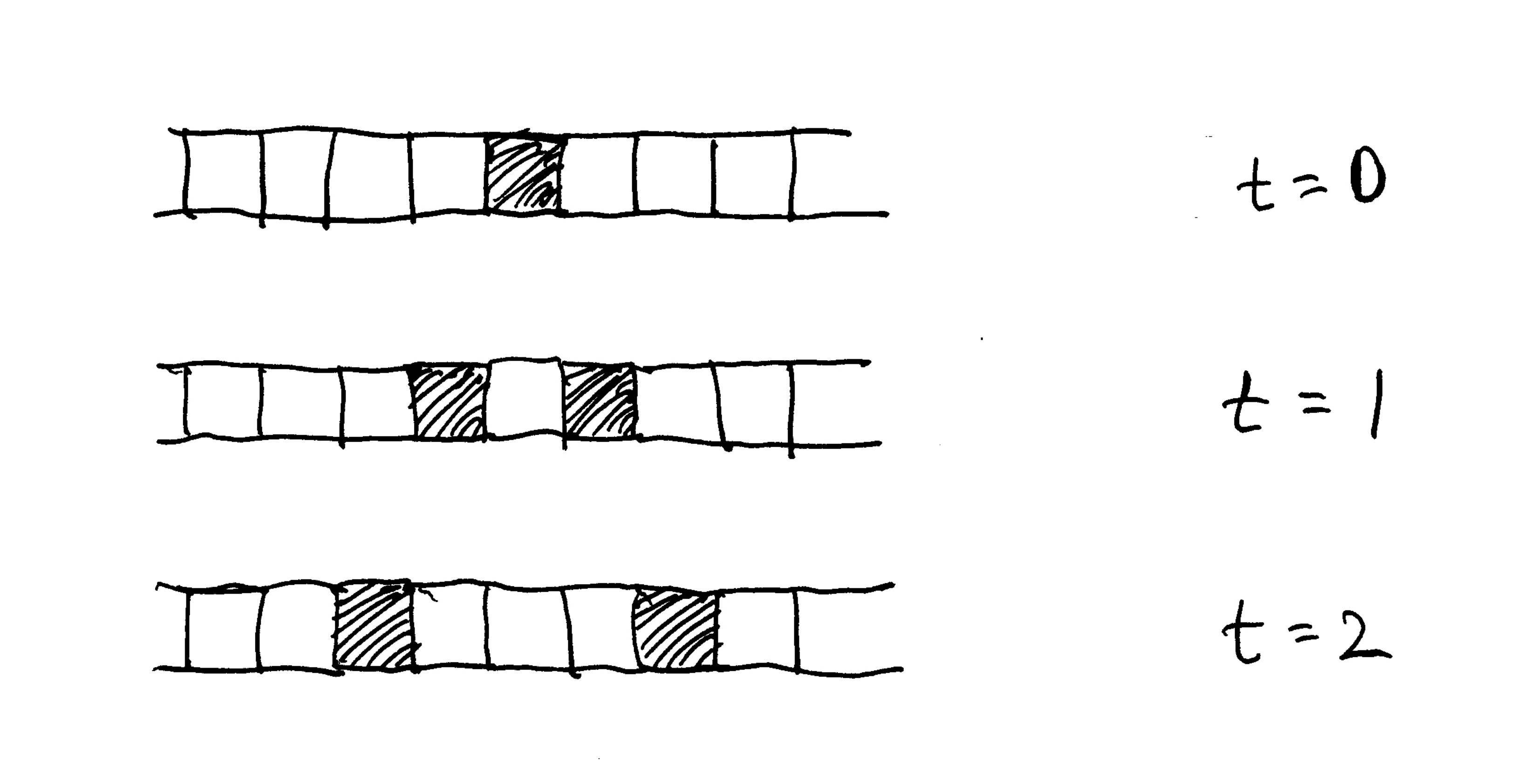
A. Cellular automata

- Arrays in which cells can assume different states
- Simplest type assume binary states: either black (on) or white (off)
- An algorithm decides how the cells change their state in discrete times
- Time: $t = 1, 2, 3, \ldots$

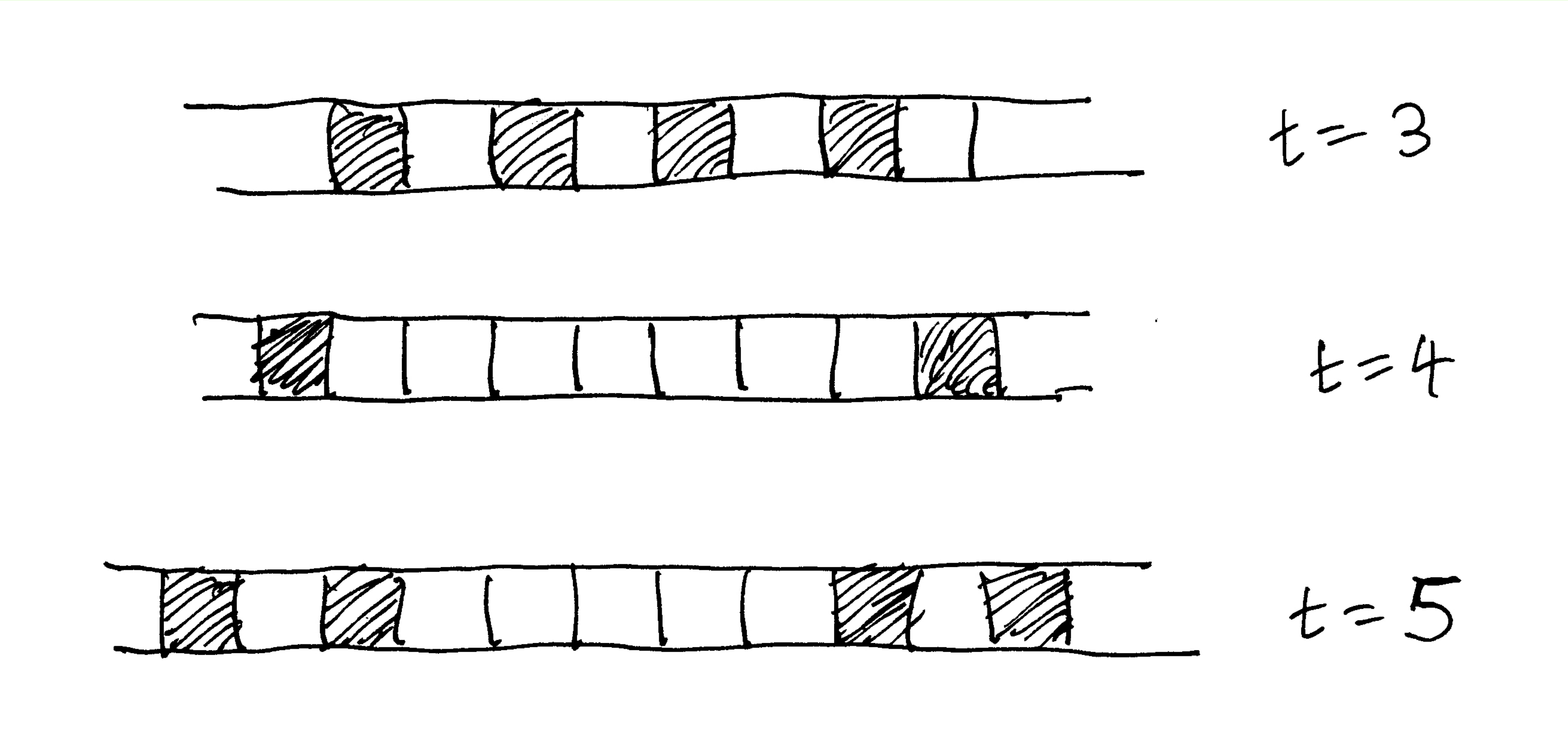
1-D cellular automata

- A line of cells
- An algorithm generates the next state
- One such rule is: "***Turn black if either neighbor is black; turn white if both neighbors are either black or white***"
- For example, begin with all states white (off) except for a single black (on) in the middle

Rule 90 — picture



Rule 90 — picture (cont.)



Not presented as design tool

- This discussion of cellular automata is directed at creating an ***analogy*** for understanding architectural design
- Not meant to be used directly
- A simple cellular automaton does not have the right complexity to be useful in adaptive design

Rule 90 formula

- Let the state of the cell at position j and at time t be $a_j(t)$
- The value of $a_j(t)$ can either be 0 or 1
- Recursive algorithm: the cell's state at time $t + 1$ is:
- $a_j(t + 1) = \{a_{j-1}(t) + a_{j+1}(t)\}$ mod2

Simpler formulation based on state of left and right neighbors

- Notation: 1 is on, 0 is off, # is either
- ***Simple rule for next state***
- 1#1 and 0#0 both become #0#
- 0#1 and 1#0 both become #1#

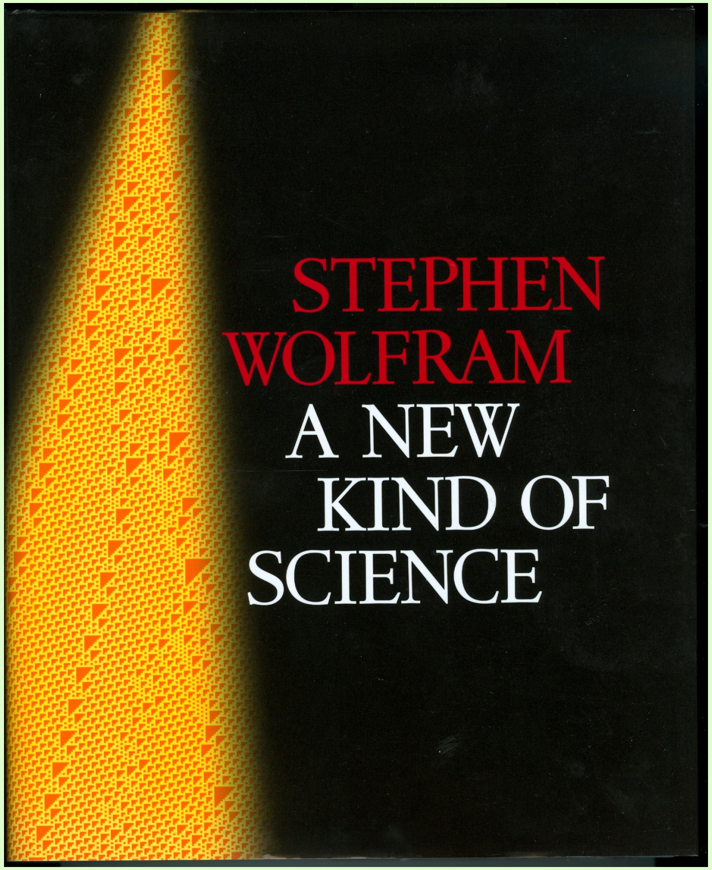
Initial condition

- Next state of a cellular automaton depends upon the previous state
- Initial conditions determine all later development
- This example began with just one black pixel (on), and the pattern grows to infinite length

Different cellular automata

- We used rule 90 in Wolfram's classification: Stephen Wolfram, "A New Kind of Science", *Wolfram Media, Champaign, Illinois, USA*, 2002.
- A different rule will define a distinct cellular automaton

“A New Kind of Science”



Nearest neighbor

- Many different types of cellular automata
- Rule 90 is a “nearest-neighbor” rule
- Simplest interaction of “on” elements — only with their nearest neighbors
- Shortest possible interaction distance
- LONG-RANGE PATTERN RESULTS FROM THIS RULE

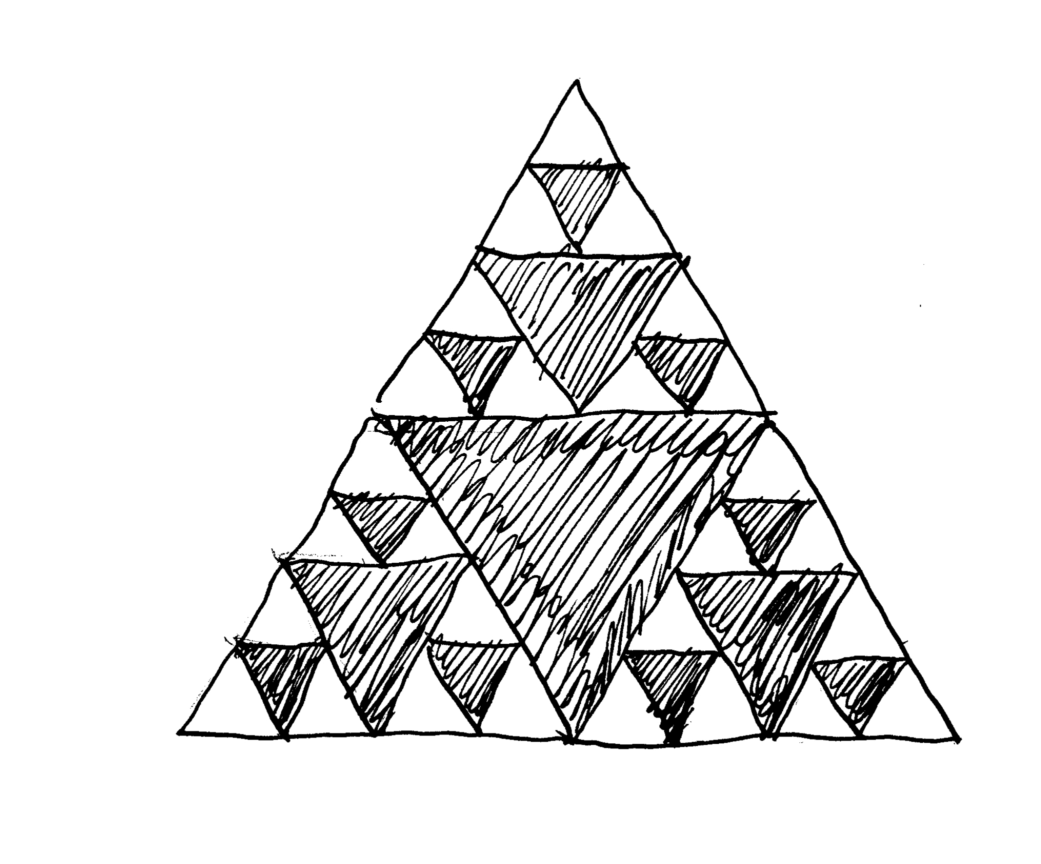
Misguided applications

- Some architects are beginning to apply Wolfram's results directly to design
- I believe they are mistaken
- Creating non-adaptive forms that look pretty, but are unsuitable for buildings
- Wolfram's cellular automata are just a set of examples useful for analogies, not for design models

B. Sierpinski carpets and sea-shells

- Cellular automaton Rule 90 generates a digitized version of the Sierpinski fractal triangle
- Different initial conditions will generate distinct fractal triangles (one is constructed later in this talk)

Sierpinski fractal triangle



Algorithmic design rules

- I am laying down the logical framework for adaptive algorithms
- Design rules should not produce a mathematical fractal, but will generate a complex structure — a building or a city — with many of the coherent features of a fractal

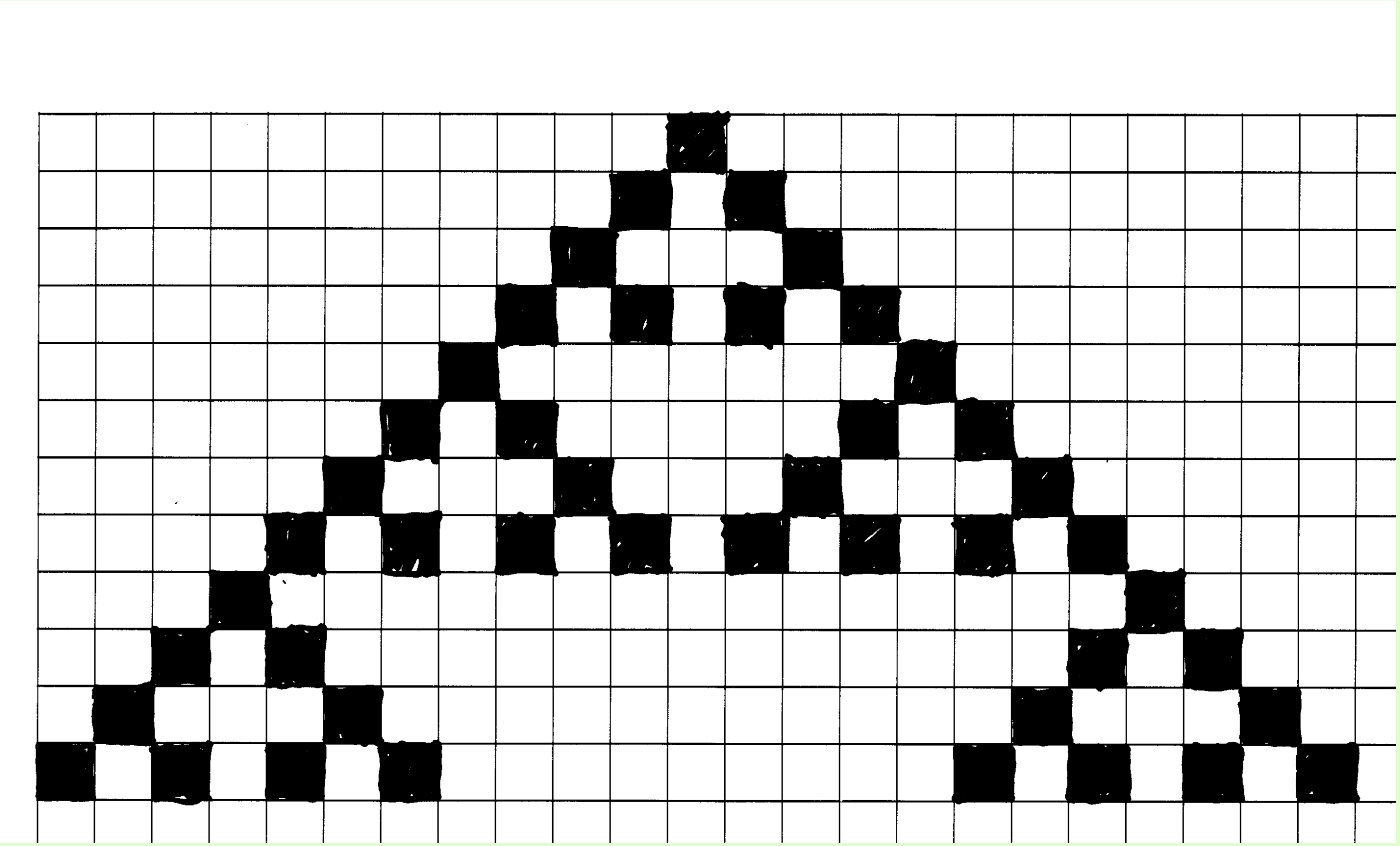
Weaving a carpet

- Human activity over Asia, the Middle East, and the entire Islamic world for millennia
- Knot one line of the carpet at a time — similar to 1-D cellular automaton
- Some cultures sing the 1-D pattern that gives each line, as it is being woven
- The result is a two-dimensional fabric

Space-time diagram

- A 1-D cellular automaton evolves in time by changing its state/appearance
- Show the time dimension of its evolution by displaying its states at different times next to each other. This results in a 2-D space-time diagram (with *x*-*t* axes)
- The diagram is a two-dimensional carpet

Sierpinski carpet



Sierpinski carpet (cont.)

- Subsequent states of 1-D cellular automaton Rule 90 “weave” the 2-D Sierpinski triangle
- Carpet is a *digitized* fractal, because there is a minimum pixel size — one cell
- As it adds more weft lines, the Sierpinski carpet gets closer to a mathematical fractal
- A perforated fractal has been created by an algorithm

Emergence of patterns

- Visual example shows “emergence”
- A recursive 1-D algorithm (on a line) involving only nearest-neighbor interactions generates a nested design — a 2-D fractal (on a plane)
- Nothing in this cellular automaton leads us to expect such ***complex long-range patterns that can be seen only in 2-D***

Architectural conclusions

- Simplest possible 1-D binary algorithm generates large-scale order
- All characteristics of coherence are present — scaling hierarchy, scaling symmetry, scaling distribution, subsymmetries, etc.
- Can we use simple rules to create great buildings and cities?
- YES! Form languages, Smart Code, etc.

Just proved an important point

- New Urbanist codes, like the Smart Code of Andrés Duany and Elizabeth Plater-Zyberk work because they generate adaptive environments
- I just showed by analogy that using the correct algorithms, it is possible to generate complex environments

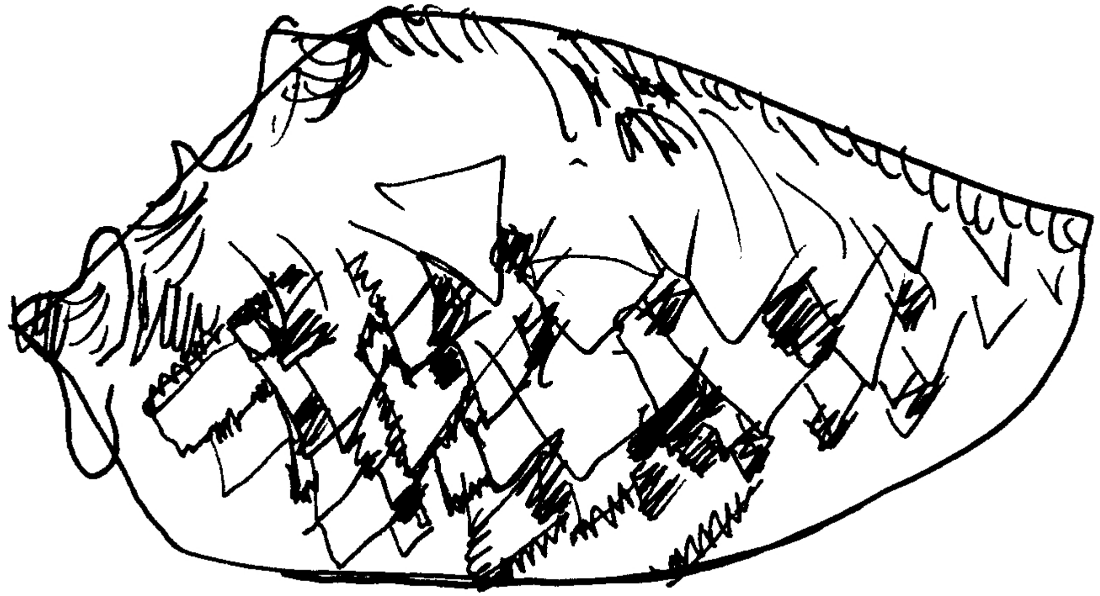
Emergence in general

- A very simple rule generates a complex pattern not explicit in the initial code
- Self-similarity, scaling coherence, and scaling distribution all arise from an algorithm acting on the smallest scale
- ***Emergent geometrical patterns are seen only in a higher dimension than the one the algorithm acts on***

First animal to apply a cellular automaton to build

- Marine mollusks generate a fractal pattern on their shells: *Tent Olive Shell* (South America), *Damon's Volute* (Western Australia), *Textile Cone* (Indo-Pacific), *Glory of the Seas* (Pacific)
- Animal lays down 1-D pattern one row at a time, as it grows the lip of its shell
- Patterns are very roughly Sierpinski-like

Seashell



Amazing

- The mollusk is growing its house using a fractal pattern — algorithmic design!
- The mollusk never gets to see the outside of its shell; it never goes out, and its eyes are not as highly developed
- While the mollusk is alive, the shell pattern is covered by an organic membrane

The Sierpinski triangle and the Binomial Theorem

- Binomial coefficients are numbers in the expansion of $a + b$ to the n-th power
- All the binomial coefficients can be computed from Pascal's triangle
- Re-compute Pascal's triangle modulo 2 (odd = 1, even = 0)
- Becomes the digitized Sierpinski triangle

Binomial expansions

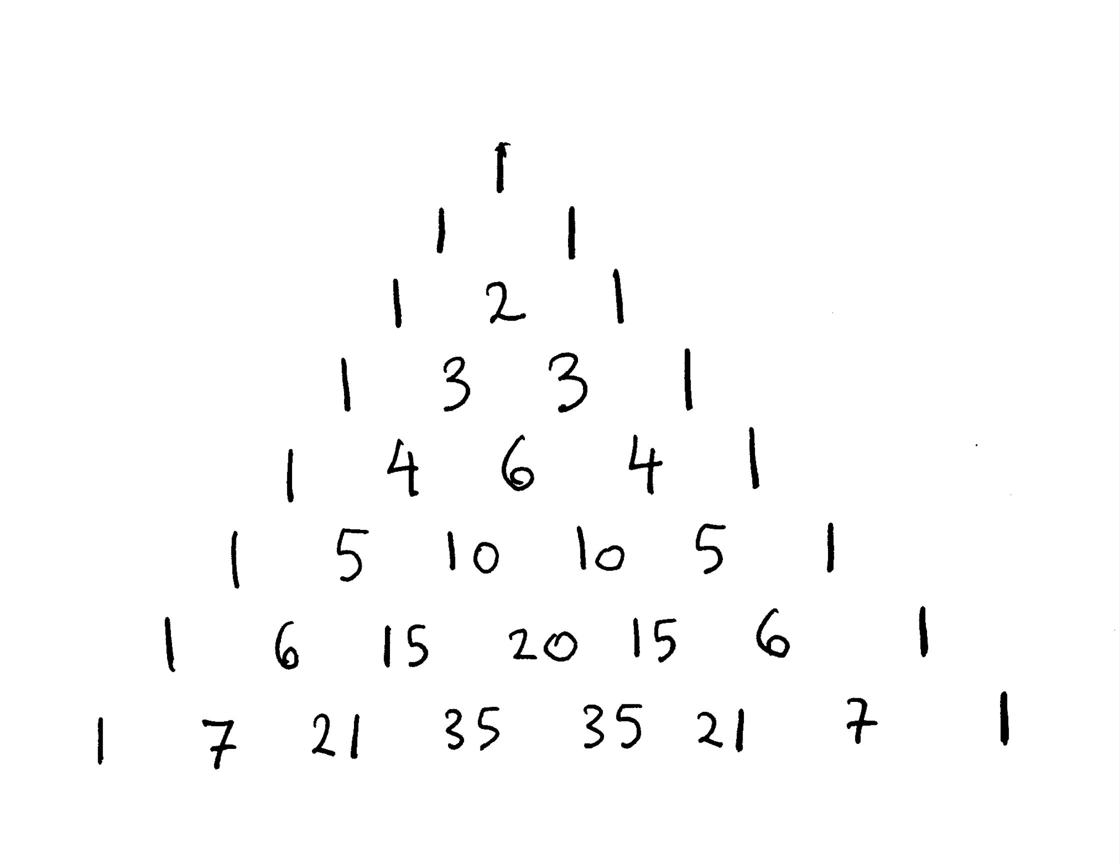
$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 =$$

$$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

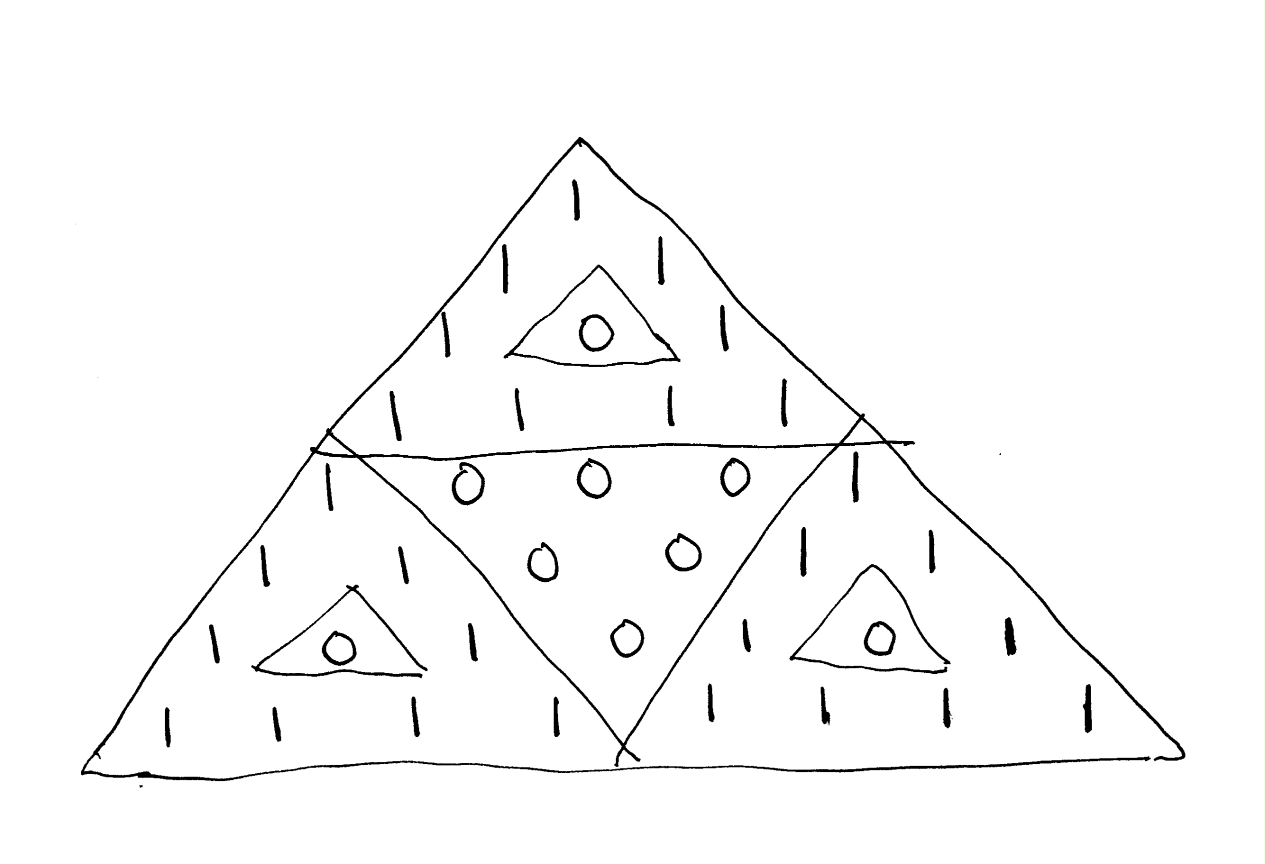
Pascal's triangle of coefficients



Simple algorithm for generating the rows of Pascal's triangle

- Begin with the zeroth power — everything equals 1
- The first power has coefficients 1, 1
- Add numbers to get 1, 1 + 1 = 2, 1
- Next line has 1, 1 + 2 = 3, 2 + 1 = 3, 1
- Continue to generate more rows…

Pascal’s triangle modulo 2 (odd = 1, even = 0) becomes Sierpinski



Classification of cellular automata

- Wolfram has classified all 256 possible 1-D cellular automata with binary states (on-off) and nearest-neighbor interactions
- Twenty of them (8%) generate variants of the Sierpinski gasket, others are not regular
- ***Generative codes are very few among all possible architectural algorithms***

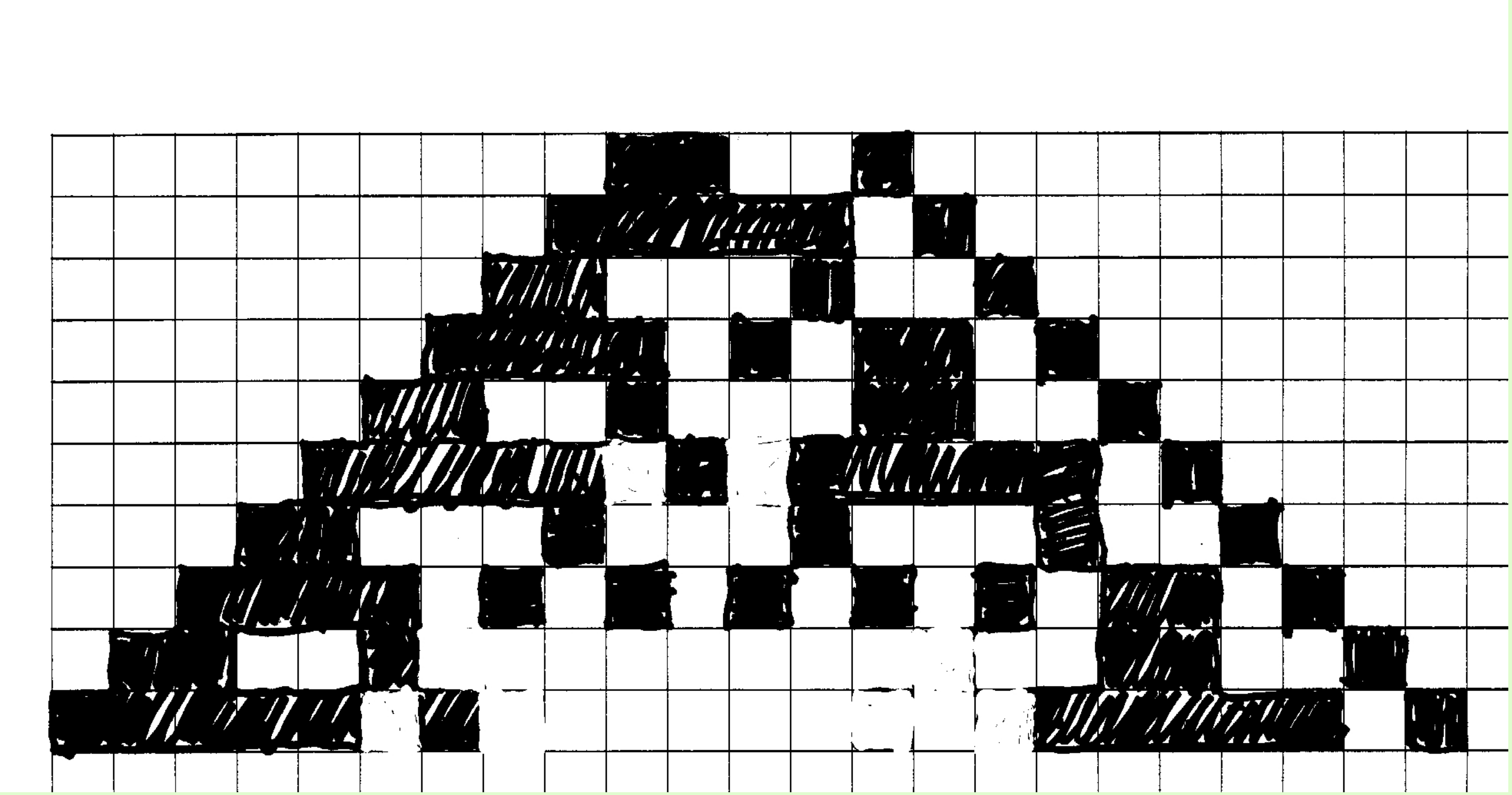
Selection of algorithms

- Even among the simplest cellular automata (nearest-neighbor, two-state systems) the majority does not generate any coherent designs!
- There are infinitely more (long-range, multi-state, etc.) cellular automata
- Rule 90 is useful because it is seen in biological structures, and is also related to the Binomial Theorem

A different initial condition

- Use Rule 90 with different initial condition
- The same cellular automaton can generate many distinct nested hierarchical patterns
- Development depends upon the initial state
- For example, begin with three black pixels (on) distributed as (11001)

Rule 90, different initial condition



Analogous implications for design

- Adaptive design is highly dependent upon initial conditions: existing structures, surroundings, human needs, etc.
- The same design algorithm will result in drastically distinct end-products
- ***The proper algorithm can be used to design buildings and cities that are each distinct because they adapt to local conditions***

Formal design is not adaptive

- Can be of either two forms:
- 1. *Non-algorithmic, which only imposes preconceived forms*
- 2. *Algorithmic but non-adaptive, not responsive to initial conditions*
- Formal designs are self-referential — they could all look the same

Algorithms in nature

- Nature only uses sustainable algorithms
- Non-sustainable algorithms die out!
- Darwinian selection based on survival
- This is *selection of algorithms* instead of *selection of forms* that we normally think of as the result of evolution

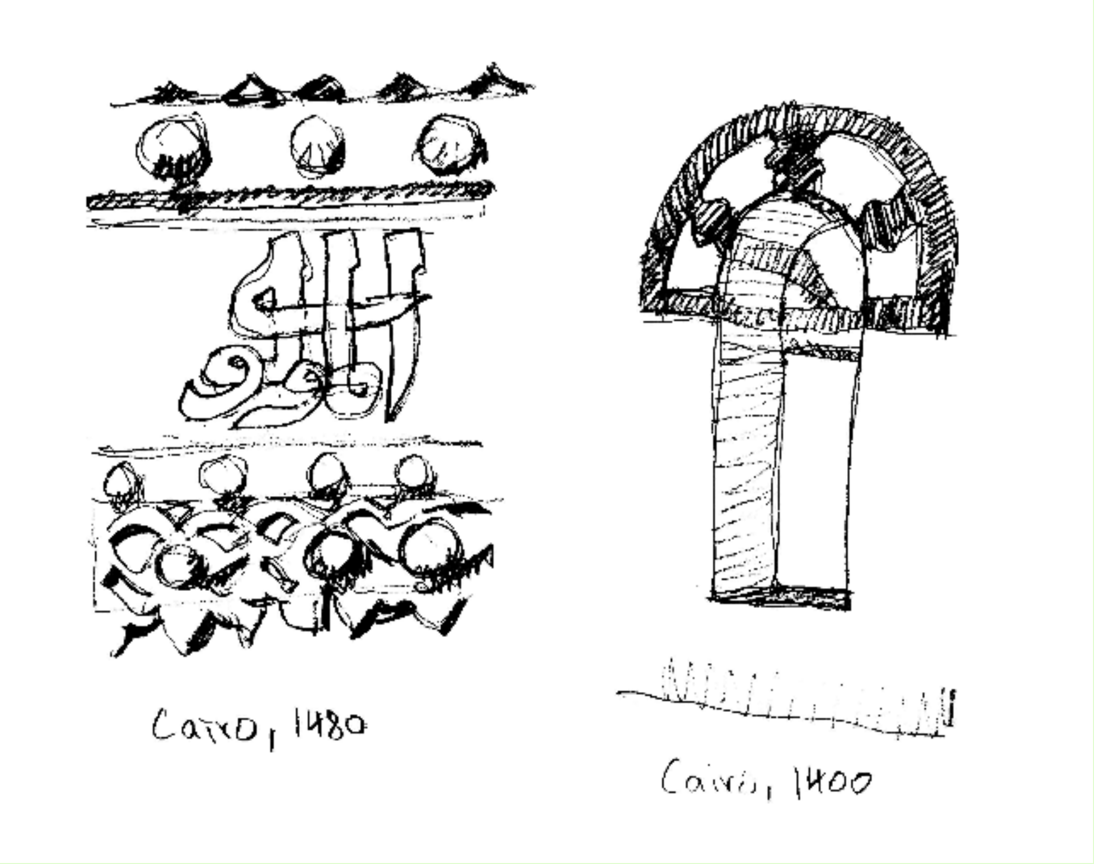
C. Design in hyperspace and connection to the sacred

- An entirely speculative direction
- Nevertheless, topic is fundamentally important to architecture
- For millennia, human beings have sought to connect to the sacred realm through architecture

Metaphysical questions

- Christopher Alexander talks about connecting to a larger coherence
- We experience this connection — a visceral feeling — in a great religious building or place of great natural beauty
- Hassan Fathy talked about the sacred structure in everyday environments

Islamic Architecture



Connecting via architecture

- Talking about connecting viscerally to a building makes people profoundly uneasy
- For millennia, our ancestors built sacred places and buildings that connect us to something beyond everyday reality
- Today's western culture does not accept this as possible

Excursions to higher dimensions

- Line — one dimension (1-D)
- Plane — two dimensions (2-D)
- Volume — three dimensions (3-D)
- In mathematics, it is perfectly normal to work in any number of dimensions
- From physics, we know that ordinary matter exists in several dimensions

Physical dimensions

- Three spatial dimensions: *x*, *y*, *z*
- Next dimensions distinguish particles
- ***Spin***: distinguishes Bosons from Fermions
- ***Isospin***: distinguishes Nucleons
- ***Hypercharge***: distinguishes shorter-lived elementary particles

Architecture in hyperspace

- Imagine a complex design or structure defined in more than 3-D
- This structure is richly patterned
- We cannot fully perceive its symmetries because of our perceptual limitations
- The only features we can see are sections of the whole *n*-D structure

Central conjecture

- ***We connect to a higher realm only through coherent complex structures***
- Coherence and symmetries of form make possible the continuation into symmetries in other dimensions
- Most 20th-Century and contemporary buildings restrict forms to 3-D or less because they are minimalist or disordered

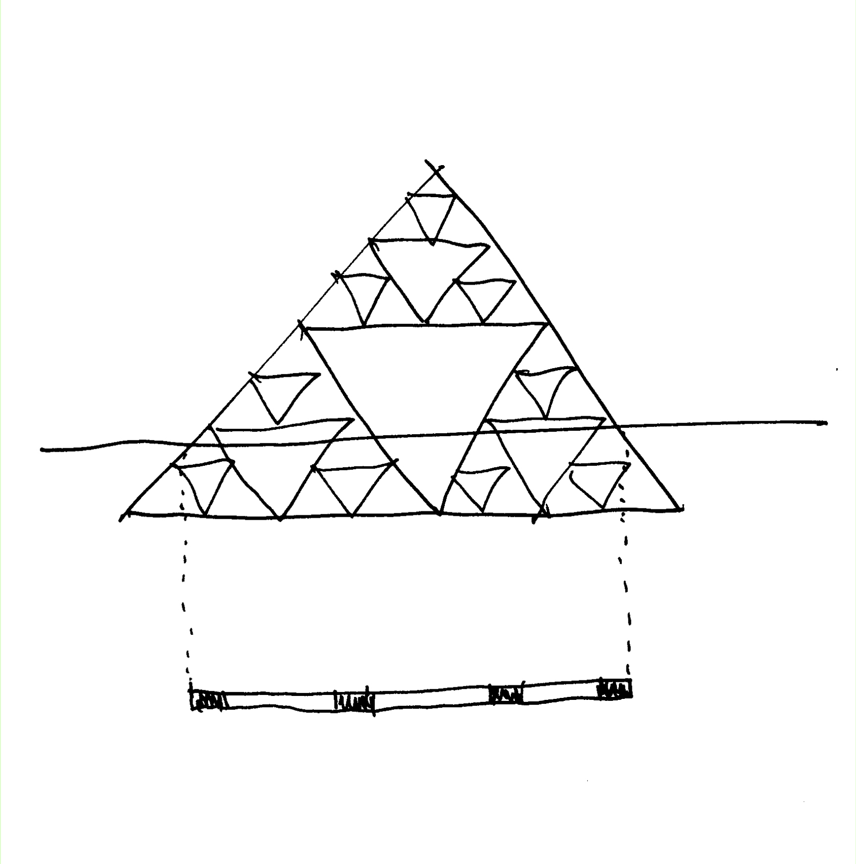
Analogy: design sections

- We used a 1-D cellular automaton to construct the 2-D Sierpinski carpet
- By analogy, people build 3-D material structures that could generate a larger coherent structure within *n*-D hyperspace
- We could thus connect to the larger *n*-D entity, which is more than what we can see

Patterns in *n*-D

- With the Sierpinski gasket, it is not possible to deduce its symmetric large-scale nested patterns from any single section
- Nevertheless, we do observe regularity in each cellular automaton with Rule 90
- ***Geometrical coherence in what we see implies a larger coherence in n-D***

Section through Sierpinski gasket



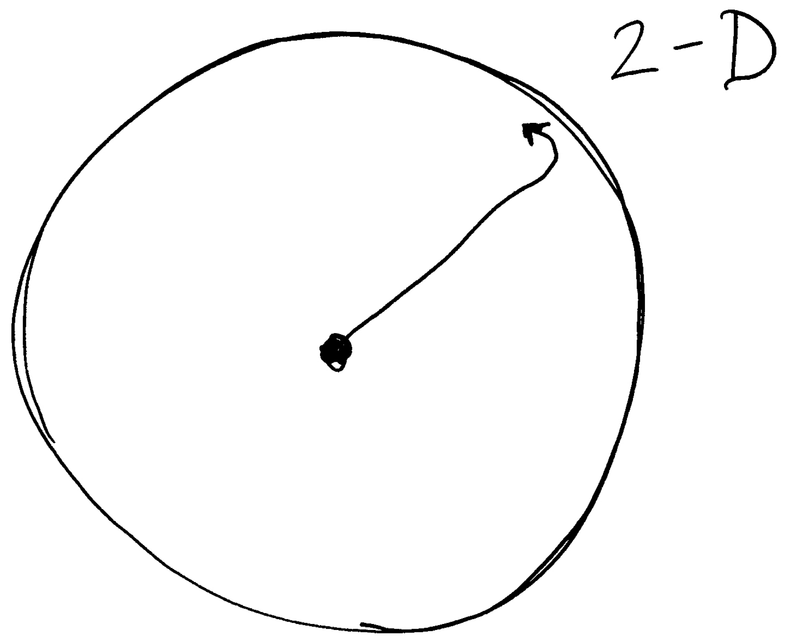
Imagined structure

- Sierpinski: patterns shown in any 1-D section imply that the original has complex, coherent structure in 2-D
- Self-similarity and scaling of the complex 2-D object show only as reduced coherent patterns on the 1-D cellular automaton

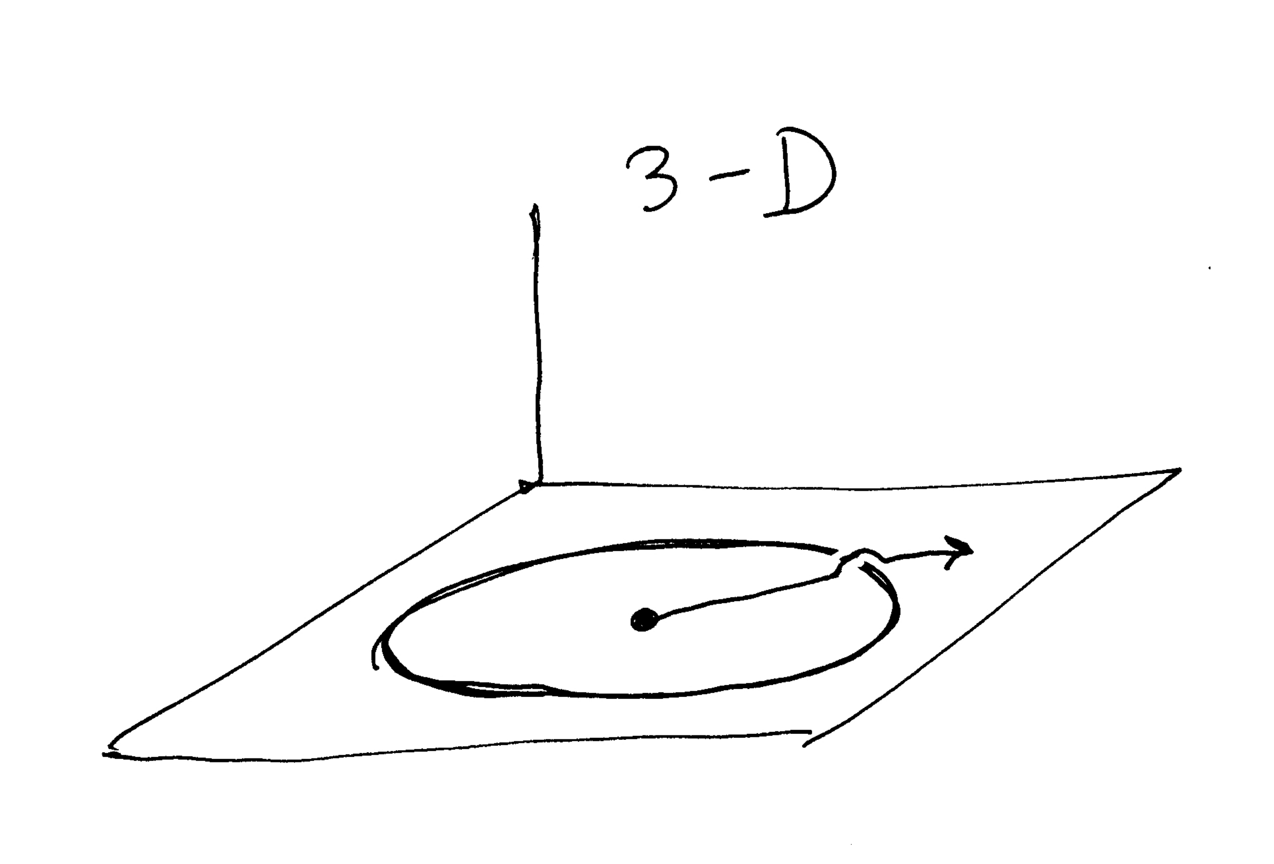
How can we connect to coherent structures in *n*-D ?

- Actually, this deeper question is easily answered with mathematics
- If we inhabit a space that is bounded, then we cannot connect to something outside it
- By going to one more dimension, we can jump over the boundary and connect
- Example: it is possible to jump in 3-D space to get over 2-D boundary

If we are bounded in 2-D …



We could jump in 3-D to get over the boundary



Philosophical/religious questions

- We have raised questions — without answering them — about connecting to a higher state of order
- How can we make a “jump” out of the physical 3-D space of buildings so as to connect to a realm beyond 3-D?
- Religions tell us that it is indeed possible

Physical/mathematical questions

- Are the additional dimensions of our existence *interior* or *exterior*?
- Spiritual approach tends to imagine a world “outside” our everyday realm
- But physics has discovered dimensions “inside” — the internal symmetries of elementary particles

Connecting

- Conjectural picture presented here highlights questions about connecting to a higher order
- Alexander addresses this topic, using empirical evidence presented in "*The Nature of Order, Book 4: The Luminous Ground*"

Limits of biology?

- How high can we rise in our emotional connection and still explain it biologically?
- Emotional highs come from love, music, art, architecture, poetry, literature
- Mechanisms of response are all biological, although the most important elements are still incompletely understood

Conditions for sacred connection

- I’m interested in geometrical, not mystical properties
- Connection is achieved through dance, music, art, and architecture
- Patterns, regularity, repetition, nesting, hierarchy, scaling, fractal structure — common feature of all

Spirituality

- Highest artistic expression is related to religion
- Bach, Mozart, Botticelli, Michelangelo; anonymous artists and architects of Islamic art and architecture, mystics of the world
- By seeking God, human beings attain highest level of connection to universe

Questions that touch on religion

- Without specifying any particular organized religion, spirituality can lead to connectivity
- Same mechanism as biophilia? Maybe — only more advanced and more intense
- Can we transcend biological connection to achieve an even higher spiritual connection?

Manifestation of the sacred

- Religious belief itself is abstract, resident in the mind
- But connection occurs through geometry, senses, music, rhythm, color
- Religious connection is very physical, oftentimes intensely so
- This physical connection gives us the materialization of sacred experience

Dance — temporal rhythm

- Bharatanatyam, classical Indian dancing
- African shamanic dance
- Native American religious dance
- Whirling dervishes in Mevlana, Turkey
- Hassidic dances
- Mystical dance forms contain geometric qualities of scaling coherence

Music — rhythm

- In the Classical West: Masses of Bach, Haydn, and Mozart
- Show fractal temporal structure — inverse power-law scaling
- Sacred chant in all religions connects
- Holy days: Byzantine Easter service, Passion Plays, Kol Nidre during Yom Kippur, Buddhist ceremonial chant

Sacred architecture

- All over the world, the House of God displays the qualities we seek to the highest possible extent
- Independent of particular religion or style
- Found among all religious building types
- Architects of the past instinctively built according to rules for scaling coherence

Conclusion

- All the examples I have mentioned have common mathematical qualities
- Fractals, symmetries, rhythm, hierarchy, scaling distribution, etc.
- Deliberate creations by humanity the world over trying to connect to something out there — or inside?