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Lecture 4

A. Cellular automata.
B. Sierpinski carpets and sea-shells.
C. Design in hyperspace and connection to the sacred.
Introduction

• Unlike the previous lectures, this lecture gives no practical model for design
• Instead, I examine a union of ideas from computer science, physics, mathematics, and spirituality
• Working from analogy, I try to get into the foundations of architecture
Relate architecture to other disciplines

• I relate the basis of architecture to other disciplines

• In the 20th Century, architecture has been isolated from the technological world and all of its impressive advances

• Sure, architects have applied technology, but they worked from an artistic basis
“Toy” models

- Scientists confronted with a highly complex problem often create a “toy model”
- Captures the essentials in a very simple model, which helps to understand the underlying mechanism
- Then work by analogy to solve the real problem
A. Cellular automata

- Arrays in which cells can assume different states
- Simplest type assume binary states: either black (on) or white (off)
- An algorithm decides how the cells change their state in discrete times
- Time: $t = 1, 2, 3, \ldots$
1-D cellular automata

• A line of cells
• An algorithm generates the next state
• One such rule is: “Turn black if either neighbor is black; turn white if both neighbors are either black or white”
• For example, begin with all states white (off) except for a single black (on) in the middle
Rule 90 — picture

$t = 0$

$t = 1$

$t = 2$
Rule 90 — picture (cont.)

\[ t = 3 \]

\[ t = 4 \]

\[ t = 5 \]
Not presented as design tool

- This discussion of cellular automata is directed at creating an *analogy* for understanding architectural design.
- Not meant to be used directly.
- A simple cellular automaton does not have the right complexity to be useful in adaptive design.
Rule 90 formula

- Let the state of the cell at position $j$ and at time $t$ be $a_j(t)$
- The value of $a_j(t)$ can either be 0 or 1
- Recursive algorithm: the cell’s state at time $t + 1$ is:
  \[ a_j(t + 1) = \{a_{j-1}(t) + a_{j+1}(t)\} \mod 2 \]
Simpler formulation based on state of left and right neighbors

- Notation: 1 is on, 0 is off, # is either

- *Simple rule for next state*
  - 1#1 and 0#0 both become #0#
  - 0#1 and 1#0 both become #1#
Initial condition

- Next state of a cellular automaton depends upon the previous state
- Initial conditions determine all later development
- This example began with just one black pixel (on), and the pattern grows to infinite length
Different cellular automata


• A different rule will define a distinct cellular automaton
“A New Kind of Science”
Nearest neighbor

- Many different types of cellular automata
- Rule 90 is a “nearest-neighbor” rule
- Simplest interaction of “on” elements — only with their nearest neighbors
- Shortest possible interaction distance
- LONG-RANGE PATTERN RESULTS FROM THIS RULE
Misguided applications

• Some architects are beginning to apply Wolfram’s results directly to design
• I believe they are mistaken
• Creating non-adaptive forms that look pretty, but are unsuitable for buildings
• Wolfram’s cellular automata are just a set of examples useful for analogies, not for design models
B. Sierpinski carpets and sea-shells

- Cellular automaton Rule 90 generates a digitized version of the Sierpinski fractal triangle
- Different initial conditions will generate distinct fractal triangles (one is constructed later in this talk)
Sierpinski fractal triangle
Algorithmic design rules

• I am laying down the logical framework for adaptive algorithms

• Design rules should not produce a mathematical fractal, but will generate a complex structure — a building or a city — with many of the coherent features of a fractal
Weaving a carpet

• Human activity over Asia, the Middle East, and the entire Islamic world for millennia
• Knot one line of the carpet at a time — similar to 1-D cellular automaton
• Some cultures sing the 1-D pattern that gives each line, as it is being woven
• The result is a two-dimensional fabric
Space-time diagram

• A 1-D cellular automaton evolves in time by changing its state/appearance
• Show the time dimension of its evolution by displaying its states at different times next to each other. This results in a 2-D space-time diagram (with $x$-$t$ axes)
• The diagram is a two-dimensional carpet
Sierpinski carpet
Sierpinski carpet (cont.)

• Subsequent states of 1-D cellular automaton Rule 90 “weave” the 2-D Sierpinski triangle
• Carpet is a *digitized* fractal, because there is a minimum pixel size — one cell
• As it adds more weft lines, the Sierpinski carpet gets closer to a mathematical fractal
• A perforated fractal has been created by an algorithm
Emergence of patterns

- Visual example shows “emergence”
- A recursive 1-D algorithm (on a line) involving only nearest-neighbor interactions generates a nested design — a 2-D fractal (on a plane)
- Nothing in this cellular automaton leads us to expect such complex long-range patterns that can be seen only in 2-D
Architectural conclusions

- Simplest possible 1-D binary algorithm generates large-scale order
- All characteristics of coherence are present — scaling hierarchy, scaling symmetry, scaling distribution, subsymmetries, etc.
- Can we use simple rules to create great buildings and cities?
- YES! Form languages, Smart Code, etc.
Just proved an important point

- New Urbanist codes, like the Smart Code of Andrés Duany and Elizabeth Plater-Zyberk work because they generate adaptive environments
- I just showed by analogy that using the correct algorithms, it is possible to generate complex environments
Emergence in general

- A very simple rule generates a complex pattern not explicit in the initial code
- Self-similarity, scaling coherence, and scaling distribution all arise from an algorithm acting on the smallest scale
- *Emergent geometrical patterns are seen only in a higher dimension than the one the algorithm acts on*
First animal to apply a cellular automaton to build

- Marine mollusks generate a fractal pattern on their shells: *Tent Olive Shell* (South America), *Damon’s Volute* (Western Australia), *Textile Cone* (Indo-Pacific), *Glory of the Seas* (Pacific)
- Animal lays down 1-D pattern one row at a time, as it grows the lip of its shell
- Patterns are very roughly Sierpinski-like
Seashell
Amazing

- The mollusk is growing its house using a fractal pattern — algorithmic design!
- The mollusk never gets to see the outside of its shell; it never goes out, and its eyes are not as highly developed
- While the mollusk is alive, the shell pattern is covered by an organic membrane
The Sierpinski triangle and the Binomial Theorem

• Binomial coefficients are numbers in the expansion of $a + b$ to the $n$-th power
• All the binomial coefficients can be computed from Pascal’s triangle
• Re-compute Pascal’s triangle modulo 2 (odd = 1, even = 0)
• Becomes the digitized Sierpinski triangle
Binomial expansions

\[(a+b)^2 = a^2 + 2ab + b^2\]

\[(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\]

\[(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\]
Pascal’s triangle of coefficients

```
  1
 1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
```
Simple algorithm for generating the rows of Pascal’s triangle

• Begin with the zeroth power — everything equals 1
• The first power has coefficients 1, 1
• Add numbers to get 1, $1 + 1 = 2$, 1
• Next line has 1, $1 + 2 = 3$, $2 + 1 = 3$, 1
• Continue to generate more rows…
Pascal’s triangle modulo 2 (odd = 1, even = 0) becomes Sierpinski.
Classification of cellular automata

• Wolfram has classified all 256 possible 1-D cellular automata with binary states (on-off) and nearest-neighbor interactions

• Twenty of them (8%) generate variants of the Sierpinski gasket, others are not regular

• Generative codes are very few among all possible architectural algorithms
Selection of algorithms

• Even among the simplest cellular automata (nearest-neighbor, two-state systems) the majority does not generate any coherent designs!

• There are infinitely more (long-range, multi-state, etc.) cellular automata

• Rule 90 is useful because it is seen in biological structures, and is also related to the Binomial Theorem
A different initial condition

• Use Rule 90 with different initial condition
• The same cellular automaton can generate many distinct nested hierarchical patterns
• Development depends upon the initial state
• For example, begin with three black pixels (on) distributed as (11001)
Rule 90, different initial condition
Analogous implications for design

• Adaptive design is highly dependent upon initial conditions: existing structures, surroundings, human needs, etc.

• The same design algorithm will result in drastically distinct end-products

• The proper algorithm can be used to design buildings and cities that are each distinct because they adapt to local conditions
Formal design is not adaptive

• Can be of either two forms:
  1. *Non-algorithmic, which only imposes preconceived forms*
  2. *Algorithmic but non-adaptive, not responsive to initial conditions*
• Formal designs are self-referential — they could all look the same
Algorithms in nature

• Nature only uses sustainable algorithms
• Non-sustainable algorithms die out!
• Darwinian selection based on survival
• This is *selection of algorithms* instead of *selection of forms* that we normally think of as the result of evolution
C. Design in hyperspace and connection to the sacred

- An entirely speculative direction
- Nevertheless, topic is fundamentally important to architecture
- For millennia, human beings have sought to connect to the sacred realm through architecture
Metaphysical questions

• Christopher Alexander talks about connecting to a larger coherence

• We experience this connection — a visceral feeling — in a great religious building or place of great natural beauty

• Hassan Fathy talked about the sacred structure in everyday environments
Islamic Architecture
Connecting via architecture

• Talking about connecting viscerally to a building makes people profoundly uneasy
• For millennia, our ancestors built sacred places and buildings that connect us to something beyond everyday reality
• Today’s western culture does not accept this as possible
Excursions to higher dimensions

• Line — one dimension (1-D)
• Plane — two dimensions (2-D)
• Volume — three dimensions (3-D)
• In mathematics, it is perfectly normal to work in any number of dimensions
• From physics, we know that ordinary matter exists in several dimensions
Physical dimensions

- Three spatial dimensions: \( x, y, z \)
- Next dimensions distinguish particles
- *Spin*: distinguishes Bosons from Fermions
- *Isospin*: distinguishes Nucleons
- *Hypercharge*: distinguishes shorter-lived elementary particles
Architecture in hyperspace

• Imagine a complex design or structure defined in more than 3-D
• This structure is richly patterned
• We cannot fully perceive its symmetries because of our perceptual limitations
• The only features we can see are sections of the whole $n$-D structure
Central conjecture

• *We connect to a higher realm only through coherent complex structures*

• Coherence and symmetries of form make possible the continuation into symmetries in other dimensions

• Most 20th-Century and contemporary buildings restrict forms to 3-D or less because they are minimalist or disordered
Analogy: design sections

- We used a 1-D cellular automaton to construct the 2-D Sierpinski carpet
- By analogy, people build 3-D material structures that could generate a larger coherent structure within $n$-D hyperspace
- We could thus connect to the larger $n$-D entity, which is more than what we can see
Patterns in $n$-D

• With the Sierpinski gasket, it is not possible to deduce its symmetric large-scale nested patterns from any single section
• Nevertheless, we do observe regularity in each cellular automaton with Rule 90
• Geometrical coherence in what we see implies a larger coherence in $n$-D
Section through Sierpinski gasket
Imagined structure

• Sierpinski: patterns shown in any 1-D section imply that the original has complex, coherent structure in 2-D

• Self-similarity and scaling of the complex 2-D object show only as reduced coherent patterns on the 1-D cellular automaton
How can we connect to coherent structures in $n$-D?

- Actually, this deeper question is easily answered with mathematics.
- If we inhabit a space that is bounded, then we cannot connect to something outside it.
- By going to one more dimension, we can jump over the boundary and connect.
- Example: it is possible to jump in 3-D space to get over 2-D boundary.
If we are bounded in 2-D ...
We could jump in 3-D to get over the boundary
Philosophical/religious questions

• We have raised questions — without answering them — about connecting to a higher state of order
• How can we make a “jump” out of the physical 3-D space of buildings so as to connect to a realm beyond 3-D?
• Religions tell us that it is indeed possible
Physical/mathematical questions

• Are the additional dimensions of our existence *interior* or *exterior*?
• Spiritual approach tends to imagine a world “outside” our everyday realm
• But physics has discovered dimensions “inside” — the internal symmetries of elementary particles
Connecting

• Conjectural picture presented here highlights questions about connecting to a higher order

• Alexander addresses this topic, using empirical evidence presented in “The Nature of Order, Book 4: The Luminous Ground”
Limits of biology?

• How high can we rise in our emotional connection and still explain it biologically?
• Emotional highs come from love, music, art, architecture, poetry, literature
• Mechanisms of response are all biological, although the most important elements are still incompletely understood
Conditions for sacred connection

• I’m interested in geometrical, not mystical properties
• Connection is achieved through dance, music, art, and architecture
• Patterns, regularity, repetition, nesting, hierarchy, scaling, fractal structure — common feature of all
Spirituality

• Highest artistic expression is related to religion

• Bach, Mozart, Botticelli, Michelangelo; anonymous artists and architects of Islamic art and architecture, mystics of the world

• By seeking God, human beings attain highest level of connection to universe
Questions that touch on religion

• Without specifying any particular organized religion, spirituality can lead to connectivity
• Same mechanism as biophilia? Maybe — only more advanced and more intense
• Can we transcend biological connection to achieve an even higher spiritual connection?
Manifestation of the sacred

• Religious belief itself is abstract, resident in the mind
• But connection occurs through geometry, senses, music, rhythm, color
• Religious connection is very physical, oftentimes intensely so
• This physical connection gives us the materialization of sacred experience
Dance — temporal rhythm

• Bharatanatyam, classical Indian dancing
• African shamanic dance
• Native American religious dance
• Whirling dervishes in Mevlana, Turkey
• Hassidic dances
• Mystical dance forms contain geometric qualities of scaling coherence
Music — rhythm

• In the Classical West: Masses of Bach, Haydn, and Mozart
• Show fractal temporal structure — inverse power-law scaling
• Sacred chant in all religions connects
• Holy days: Byzantine Easter service, Passion Plays, Kol Nidre during Yom Kippur, Buddhist ceremonial chant
Sacred architecture

• All over the world, the House of God displays the qualities we seek to the highest possible extent
• Independent of particular religion or style
• Found among all religious building types
• Architects of the past instinctively built according to rules for scaling coherence
Conclusion

• All the examples I have mentioned have common mathematical qualities
• Fractals, symmetries, rhythm, hierarchy, scaling distribution, etc.
• Deliberate creations by humanity the world over trying to connect to something out there — or inside?