

APPLICATIONS OF THE GOLDEN MEAN TO ARCHITECTURE

Nikos A. Salingaros, The University of Texas at San Antonio.

Abstract: *The Golden Mean, an irrational number related to the Fibonacci sequence, arises in the study of biological growth and hierarchical systems. Quite distinct from natural structures that exhibit such growth patterns, artists and architects have long made extraordinary assertions about a preference for rectangles having aspect ratio 1.618:1 approximating the Golden Mean. Such claims are false and are chiefly due to failing to measure things accurately. These embarrassing errors are perpetuated by a kind of cult mysticism. The Golden Mean does apply to architectural composition in the context of scaling hierarchy that organizes complexity; this design methodology is unrelated however to a rectangle's aspect ratio. Simply considering rectangular aspect ratios does not guarantee good design.*

1. Introduction.

The architectural literature frequently mentions an irrational number called the Golden Mean. It is common to claim that the Golden Mean has influenced the shape of countless important buildings throughout history, and that it is somehow responsible for any “balance” and “harmony” that they possess. Supposedly, a rectangle with aspect ratio satisfying the Golden Mean has special, almost magical, properties. Even though there are numerous instances of the Golden Mean arising in natural systems (in growth patterns, not in rectangles), it is problematic to find unambiguous examples of Golden Mean rectangles in man-made structures (Livio, 2003). This is surprising, considering the vigorous claims made by so many authors convinced otherwise.

The Golden Mean does indeed arise in architectural design, as a method for ensuring that a structure possesses a natural and balanced hierarchy of scales. In this practical application, the Golden Mean plays the same — experimentally verified — role found in a wide variety of structures in nature that exhibit hierarchical scaling. Nevertheless, this result has nothing to do with a rectangle's aspect ratio, which is the only topic the majority of proponents of the Golden Mean in architecture want to focus on.

Two experimental results by George Markowsky in fact disprove the main claim for

Golden Mean rectangles. It turns out that: 1. *People do not prefer a rectangle with aspect ratio the Golden Mean to other rectangles*; and 2. *People cannot even identify a Golden Mean rectangle when it is placed amongst other rectangles!* (Markowsky, 1992).

The present paper reviews three paradigmatic case studies where Golden Mean rectangles allegedly apply in architecture: (i) The Parthenon in Athens; (ii) the Villa Stein/Garches by Le Corbusier outside Paris; and (iii) the United Nations Building in New York City. None of these is an example of the Golden Mean in the way usually claimed, a result that was already anticipated by Christopher Alexander (Alexander, 1959) and Rudolf Wittkower (Wittkower, 1960). Marco Frascari and Livio Volpi-Ghirardini reject the claimed applications of Golden Mean rectangles to architecture because proponents never measured the buildings in question (Frascari & Volpi-Ghirardini, 1998).

So, why do architects continue to uphold historic misconceptions about the Golden Mean? The reason is that people will believe myths while remaining oblivious to both mathematical proof and scientific experiments. Moreover, this problem is not confined to a small group of Golden Mean enthusiasts looking for arcane mathematical relationships in historical buildings. It surfaces, for example, in major contemporary projects that try to use the Golden Mean as one of their selling points. This tactic, however, has nothing to do with either Mathematics or the Golden Mean, but is instead symptomatic of a separation of architecture from science, and an intellectually dishonest approach to the discipline by practitioners eager for commissions.

Alexander and other researchers, including myself, are investing considerable effort to determine ways of enhancing the quality of life through informational nourishment from our surroundings (Alexander, 2010; Salingaros, 1997; 1998; 2010). We would like to apply universal mechanisms responsible for positive effects in architecture. Ever since the beginnings of civilization, people have modeled natural complexity in order to understand the basis of beauty. Mixing religion, science, and art gave birth to early scientific and artistic advances. This project was derailed however, after natural complexity as our inspiration was replaced by an abstracted simplicity. Symptomatic of this loss is a focus on an inessential detail, such as the Golden Mean rectangle, which is then used to replace natural complexity.

2. The Golden Mean and the Fibonacci sequence.

What is called the Golden Mean (or Golden Ratio) is an irrational number approximately equal to 1.618 and usually denoted ϕ (the Greek letter Phi). This number arises as the solution to the problem of subdividing a rectangle into a square x^2 and a remaining, smaller rectangle that is similar to (i.e. has the same aspect ratio as) the original large rectangle (Figure 1).

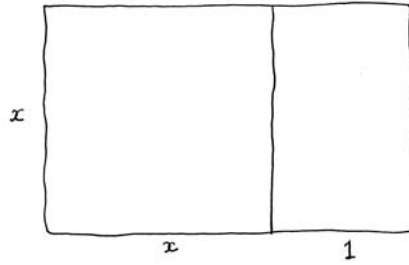


Figure 1. A Golden Mean rectangle in which the piece “left over” after defining a square has the same aspect ratio as the original rectangle.

The geometrical problem is described by the relation $(x + 1)/x = x$, leading to the equation:

$$x^2 = x + 1 \tag{1}$$

Equation (1) has the positive exact solution $x = \phi = (1 + \sqrt{5})/2$, which is the Golden Mean. The presence of a square root of a number that is not a perfect square makes this number irrational.

There is a link between the Golden Mean and the Fibonacci sequence. Consider the sequence of positive integers $\{ a_n \}$ defined by the recursion relation $a_{n+2} = a_{n+1} + a_n$, with $a_1 = a_2 = 1$, giving:

$$\{ 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, \dots \} \tag{2}$$

This sequence is the simplest way to describe growth by adding two numbers to obtain the next one. Starting from 1 and 1 generates the entire sequence.

One can obtain rational approximations of the Golden Mean ϕ with ever increasing accuracy from the ratios of consecutive numbers in the Fibonacci sequence Equation (2), for example $8/5$, $13/8$, $21/13$, etc. This result is due to the great astronomer Johannes Kepler (Livio, 2003: p. 152). Listing some of these ratios gives the following approximations:

$$\{ \dots 8/5 = 1.60, 13/8 = 1.625, 21/13 \approx 1.615 \dots \}$$

Therefore, the ratio of successive terms in the Fibonacci sequence Equation (2) tends to the Golden Mean ϕ in the limit:

$$a_{n+1} / a_n \rightarrow \phi = 1.6180339887\dots$$

3. Hierarchical subdivisions and scaling.

Any serious theory of architectural design ought to describe hierarchical subdivisions, scaling, and the relationship among distinct scales. I propose that a building should have well-defined subdivisions at dimensions that correspond to powers of e , Euler’s constant (or the base of natural logarithms), equal to 2.7182818284... (Salingaros, 1998; 2010). The design method uses the largest dimension L of the structure, making sure that substructures exist very roughly at L/e , L/e^2 , L/e^3 , etc. all the way down to the size of the granulation in the materials themselves (see Figure 2, below). The dimension (size) of components at each of these levels of scale is approximate; what is crucial is that no

level of the hierarchy should be missing.

Design is linked mathematically with natural growth through hierarchical subdivisions at distinct scales, which is found in a majority of natural structures. There is furthermore a regular geometrical relation among different scales of substructure, and in many cases, the scales are related by a single scaling factor. This theory is based on systematic observations and measurements by Christopher Alexander, who found that scaling factors of around 3 (with an extended range roughly between 2 and 5) tend both to predominate in nature, and to be preferred by human observers (Alexander, 2001).

A crucial lesson that comes from understanding natural structure is to realize that scales in a natural hierarchy are skewed towards the smallest sizes. Natural growth begins at the infinitesimal scale and develops through an ordered hierarchy up to the largest size. The spacing of different scales is therefore not uniform. There are proportionately more small levels of scale than large scales (Alexander, 2001; Salingaros, 1998; 2010), something that is not obvious from discussions about the size of the scales themselves. I will explain this later.

If one wants to recast this scaling theory as a sequence of integer factors so as to compare it to the Fibonacci sequence Equation (2), then successive powers of e can be rounded out to:

$$\{ 1, 3, 7, 20, 55, 148, 403, 1097, \dots \} \quad (3)$$

This sequence makes more accurate an old prescription sometimes used in traditional design: “*subdivide everything by three*” (Figure 2). Of course, all that a designer needs is to repeatedly divide by $e \approx 2.718$, and most pocket calculators have e built in. The numbers in the sequence (3) have no intrinsic importance: they simply approximate an exponential sequence of scales by integers for the purpose of comparing with Equation (4), below.

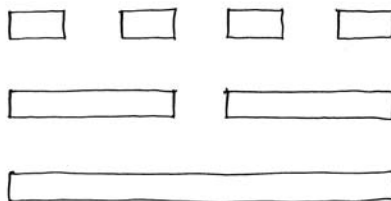


Figure 2. Components of a complex structure have sizes 1, 3, 7, etc. that follow from the integers which round off the exponential sequence.

To clarify the point about scales being distributed more towards the smaller end of the spectrum, let's generate a set of measures as a guide for some design project, beginning from the smallest perceivable detail at, say, 0.5 mm. Multiplying repeatedly by the scaling factor e gives the following example sizes, where the numbers are rounded off:

$$\{ 0.5 \text{ mm}, 1 \text{ mm}, 4 \text{ mm}, 10 \text{ mm}, 3 \text{ cm}, 7 \text{ cm}, 20 \text{ cm}, 55 \text{ cm}, 1.5 \text{ m}, 4 \text{ m}, 11 \text{ m} \}$$

In actual design, the brief and human dimensions fix the larger scales, then the smaller scales are computed from those: here we worked in the opposite direction —

from small to large — in order to illustrate the theory. These measurements may be useful or not depending upon whether the larger sizes correspond closely to what a particular design requires. Note that in a structure of 11 m size, there are eight scales smaller than, and only three scales larger than 1 m. This is a key to understanding the enormous discrepancy between traditional and modernist design (Alexander, 2001; Salingaros, 1998; 2010).

The sequence of integer approximations to the powers of e in Equation (3) compares very roughly to alternate terms of the Fibonacci sequence Equation (2). That means that the even terms of the Fibonacci sequence Equation (2) could, if desired, be used for the theory of design based upon a scaling hierarchy (Alexander, 2001; Salingaros, 1998; 2010):

$$\{ 1, 3, 8, 21, 55, 144, 377, 987, \dots \} \quad (4)$$

The numbers in Equation (4) can be generated as a sequence $\{ b_n \}$ with recurrence relation $b_{n+2} = 3b_{n+1} - b_n$, $b_1 = 1$, $b_2 = 3$. In the limit, ratios of consecutive terms of Equation (4) tend to a number $\psi = 2.6180339887\dots$ (the Greek letter Psi), which is the positive solution of the equation $x^2 = 3x - 1$. This number ψ is related to the Golden Mean as $\psi = \phi^2 = \phi + 1 \approx 2.618$, and notice that ψ is within 4% of the value of e . All of this discussion attempts to make more useful Alexander’s original findings of scaling by a factor anywhere from 2 to 5 (Alexander, 2001).

Design can thus be guided by knowing a sequence of sizes that should be defined very approximately by the tectonics of the structure itself. Where structural members don’t provide a required scale, the architect creates ornament. This is a key point. To the best of my knowledge, architects have never consciously implemented this tool (other than sometimes applying the “rule of 3”), yet we universally find built examples with such subdivisions. What probably happened is that builders throughout history simply created subdivisions that “felt right” because those mimicked natural forms.

Golden Mean enthusiasts will find much in common here with the proposed design theory, as can be seen from the nested rectangles shown in Figure 3. This diagram illustrates two important features. First, it generates a hierarchy of scales — not ratios of sides — that can then be used to approximately regulate a structure’s subdivisions. The second point is to recognize fractal scaling, where similar components (four rectangles in this simplified case, but in practice any shape at all) repeat at different magnification. Scaling similarity is the main characteristic of all fractals, and can be found in many of the world’s most beloved historical buildings (Alexander, 2001; Salingaros, 1998; 2010).

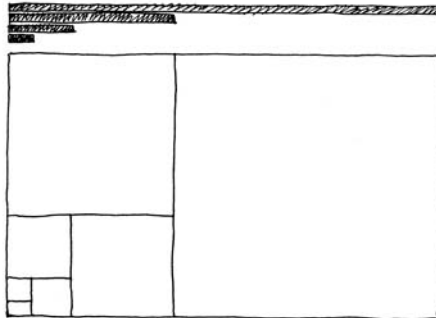


Figure 3. One could equally well use nested Golden Mean rectangles to generate the alternate Fibonacci numbers, which then define the sizes in a natural scaling hierarchy.

The relative lengths in Figure 3 correspond to the numbers in Equation (4). Why do we want to use only every other term of the Fibonacci sequence? The reason is that we wish to measure and compare the size of design components using a *scaling ratio*, and not compare sides of a rectangle that define an *aspect ratio*.

To summarize: the Golden Mean ϕ is useful in human creations in the same way it is found to occur in nature, where it related to the hierarchical scaling that is a consequence of organic growth. Any design that is meant to appeal to human users in the same mathematical sense as complex structures ought to exhibit a natural hierarchy of scales, and those can be generated by either an exponential or a Fibonacci sequence. A scaling theory provides a checklist of component sizes that together define a complete, natural scaling hierarchy. Scaling is the key feature in a proportional system: the ratio between consecutive sizes tends to $\psi = \phi + 1$, and is not the same as the ratio of sides ϕ .

The design method just described assumes that built structures will be approximate, and thus allows for a wide tolerance and considerable deviations from the numbers given above. Thus, in real-world design, one creates an approximate hierarchy of different scales in trying to mimic natural growth as best as possible: not through precision, but through hierarchical complexity. The simplistic application of Golden Mean rectangles that I'm now going to criticize, on the other hand, is based on the irrational property of the number ϕ . With minimalist rectangles, therefore, the imagined effect depends entirely upon an unrealistic degree of accuracy that is impossible to achieve in practice.

4. Aesthetic response to a Golden Mean rectangle.

The widely-circulated claim that a rectangle with aspect ratio ϕ gives maximum visual and aesthetic pleasure to an observer is false. While the idea of optimizing our positive psychological response from a design is sound, here we find a serious disagreement with experiment. Surveys carried out by George Markowsky revealed that people most commonly select a rectangle with aspect ratio 1.83:1 out of 48 rectangles with aspect ratios ranging from 1:1 (square) to 2.50:1 (Markowsky, 1992).

Even this result is further complicated, however, by psychological experiments performed by Michael Godkewitsch, who found that subjects tend to prefer the average of the sample presented to them, regardless of the actual aspect ratios (Godkewitsch, 1974). Thus, it may be possible to obtain any result desired as long as one chooses the range of different rectangles in the experiment. Godkewitsch concludes, therefore, with: *"The basic question whether there is or is not, in the Western world, a reliable verbally expressed aesthetic preference for a particular ratio between length and width of rectangular shapes can probably be answered negatively. Aesthetic theory has hardly any rationale left to regard the golden section as a decisive factor in formal visual beauty."* (Godkewitsch, 1974).

Earlier, one of the West's most eminent architectural historians, Rudolf Wittkower (Wittkower, 1960), and one of our generation's most distinguished architects and thinkers, Christopher Alexander (Alexander, 1959), simultaneously and independently disproved nineteenth-century claims that the Golden Mean is responsible for a unique aesthetic experience. Wittkower cites Theodor Lipps, who already in 1903 published

results from experimental aesthetics showing that: “*the ratio of the Golden Section, generally and in the case of the Golden Rectangle, is entirely without aesthetic significance in itself, and that the presence of this numerical ratio is nowhere the basis of any pleasant sensation.*” (Wittkower, 1960). Visual experiments establish that people can’t distinguish between rectangles with ratio the Golden Mean ϕ and one whose aspect ratio differs by up to 6% (Alexander, 1959) (Figure 4).

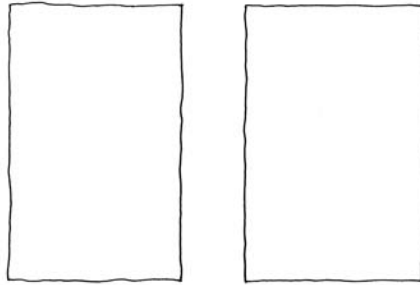


Figure 4. Is the rectangle on the right more pleasing? The Golden Mean rectangle is the one on the left.

One can trace claims for the alleged human preference for Golden Mean rectangles to now discredited experiments from the 1860s performed by Gustav Fechner (Godkewitsch, 1974; Markowsky, 1992; Wittkower, 1960). We have here a classic case of a non-reproducible experiment. Just because someone makes a numerological argument in support of an idea does not make it right: the essence of modern science is to perform experiments to prove or disprove such claims. None of the numerous authors who trustingly refer to Fechner’s claims have checked his experimental procedure, which not only was too limited to have any validity, but also was unconsciously rigged to give the intended result. Every recent experiment designed to empirically demonstrate a preference for the Golden Mean rectangle has provided a negative result (Godkewitsch, 1974; Markowsky, 1992).

5. The Golden Mean and modernist architecture.

Modernism removed traditional design elements that defined ordered complexity, such as hierarchical substructure and ornament at the human scale (Alexander, 2001; Mehaffy & Salingaros, 2002; Salingaros, 1997; 1998; 2010). But this is precisely the mathematical sense in which nature manifests Fibonacci growth and the Golden Mean in the many cases that have been discovered (Livio, 2003). Without hierarchically-organized complexity there is almost nothing left to analyze mathematically. The smaller scales that generate the larger scales through organic growth are entirely absent, leaving the largest scales mathematically detached. Human beings perceive this as unnatural.

The fascination with Golden Mean rectangles as abstraction that replaces natural complexity is intimately linked to the industrialization of the architectural aesthetic implemented by the Bauhaus. A key propagandist for Golden Mean rectangles was Ernst Neufert, an original member of the Bauhaus, who in 1939 was appointed by Albert Speer to head a study on the standardization of German industrial architecture via modular

production. Neufert published three books during (and by) the Nazi regime about the Golden Mean in the context of standardized spaces and dimensions for architecture (Frings, 2002). Another propagandist for Golden Mean rectangles is Le Corbusier, who will be discussed later in this paper.

Modernist architects allowed almost no mechanisms that could help the user connect to a building. A minimalist building feels alien precisely because it lacks mathematical information that our cognitive system uses to analyze the natural environment (Alexander, 2001; Mehaffy & Salingaros, 2002; Salingaros, 1997; 1998; 2010). A smooth rectangular slab presents information only in its dimensions, and very little else. Minimalism focuses one's attention on a single mathematical relationship — the rectangle's aspect ratio. The Golden Mean became popular among modernist architects as one of the few techniques left, as they tried desperately to make their empty rectangular slabs likeable. Seizing on its mythical attractiveness, they thought that the Golden Mean could save them after they had eliminated all emotionally connective architectural components.

A significant philosophical shift enabled modernism and its “Geometrical Fundamentalism” to spread in the first place: it appealed to the architect's intellect rather than to the user's senses (Mehaffy & Salingaros, 2002). Contrast the visceral, sensual pleasure of traditional, vernacular, and Art Nouveau Architectures with their rich curves and colors inspired by nature, against the stark “machine-aesthetic” minimalism of modernist buildings. Focusing on a strictly intellectual point — such as a mysterious irrational number with a history of mystical properties — helped to distract people from the fact that human sensory experience had been suppressed in modernist architecture.

I have often heard this claim: “*architecture progressed in the early 20th Century by incorporating the message of older visual ornamentation into its intrinsic dimensions, thus modern architecture works just as well but in a different way that is intellectually superior*”. It does nothing of the sort. Hierarchical scaling, ordered substructure, and ornament, which are essential for generating coherent complexity and adaptation in architecture, were not transformed: they were eliminated with a vengeance. The idea that the modernist idiom somehow transmutes essential complex information encoded as proportions is a deliberate falsehood meant to sell a minimalist geometry. Visual emptiness can never relate to human feelings. And yet people have fallen victims to this deception.

Nowadays, the Golden Mean continues to be misused for its purely mystical value, to sell monstrous or otherwise bizarre structures that fail to connect emotionally with the user. An imagined intellectual property is claimed to make the design attractive (even in cases where it is impossible to see an aspect ratio near the Golden Mean). Non-mathematicians such as clients, politicians, and architecture critics are impressed and perhaps intimidated by words such as “irrational number”, “Golden Mean”, “fractal”, “nonlinear”, etc., which are sprinkled throughout publicity releases arguing why some contemporary project represents a “visionary” building.

6. Proportional ratios.

Historians have long known that architects relied, as a practical expedient and during many centuries of building activity, on the smallest integers and their ratios. Going back to the Roman architectural theorist Vitruvius, we find formulas for using ratios of

integers in modular systems of measurement to aid design. For any rectangle that appears in a man-made structure, a decision has to be made on its aspect ratio. It is easy to find simple aspect ratios such as 2:1, 3:1, 3:2, 5:3, 8:5, 9:4, etc. in many surviving buildings. Other practical ratios, this time approximations to irrational numbers, include $\sqrt{2} \approx 1.41$, the diagonal of a square of side 1, and $\sqrt{3} \approx 1.73$, the bisector of an equilateral triangle of side 2. Both of these are useful because they are easily traced.

People have measured built rectangles with aspect ratio 8:5, whether on building façades or plans, and assumed that they must represent the Golden Mean. One cannot tell the difference in a building (8:5 = 1.60:1 differs by 1.1% from ϕ), but that is not the point. Many Golden Mean enthusiasts consider the mystical essence of the Golden Mean to reside in its irrationality, thus if a rational number is substituted, this essence is lost. A rectangle with proportional ratio close to 8:5 has none of the mystique attributed to the Golden Mean ϕ , because 8/5 is rational, whereas ϕ is irrational. Even if one could measure an aspect ratio of 809:500 = 1.618:1 in actual buildings, this much better approximation to the irrational Golden Mean is still rational.

In the words of Alexander: “*the irrational numbers make no sense as physical lengths*” (Alexander, 1959). A physical length is accurate only up to how closely it can be measured, and could not possibly represent a non-repeating decimal. The mystique of an irrational aspect ratio ϕ exactly built into some building makes sense only when something is measured to infinite accuracy. That’s impossible. Built examples that are touted as obeying the Golden Mean are sufficiently irregular as to give a significant error, thus making claims based on nonexistent precision impossible to justify.

7. Credit cards, movies, cameras, and computer screens.

None of the rectangles characteristic of modern technology, though built to very exact tolerances, match the Golden Mean, and nobody minds that. There exists a popular misconception that credit cards are manufactured according to the Golden Mean. It cannot be said that credit cards are intended to accurately represent Golden Mean rectangles, because their aspect ratio is defined by the ISO/IEC 7810 standard as 85.60 mm x 53.98 mm hence 1.5858:1, which differs by 2% from the Golden Mean.

Much of the earth’s population is today staring at computer, film, or television screens during many of its waking moments. One would think that the television, film, and computer industries try to utilize a human preference for a specific aspect ratio. But there is none. Various aspect ratios have been used for computer screens, including the unofficial but ubiquitous standard of 4:3 $\approx 1.33:1$. The *MacBook Pro* computer’s 15-inch screen does have 1,440x900 pixels, and thus an aspect ratio of 8:5 = 1.60:1. This is within 1.1% of the Golden Mean. Had the perfectionist Steve Jobs wished to use the Golden Mean, he could have easily used 1,440x890 pixels for an aspect ratio of 1.6180:1, but did not. And the new 11-inch *MacBook Air* has a 1,366x768 pixel screen with aspect ratio 1.78:1.

The Universal Video Format used for standard television since its inception in the 1930s has aspect ratio 4:3 $\approx 1.33:1$ (originally defined by Thomas Edison for films in the late 19th Century, adopted by the Society of Motion Picture Engineers in 1917 as its first standard, and continued into television as SDTV). Nowadays, however, the new standard for HDTV screens is the “sixteen-to-nine” aspect ratio 16:9 $\approx 1.78:1$. In the 1930s, films modified their aspect ratio to the Academy Standard of 1.37:1. The commonest format today for films shown in theaters is the American Standard Widescreen (Academy Flat)

aspect ratio of 1.85:1.

Ever since the first *Leica* cameras started using 35 mm film for still pictures in the early 20th Century, the standard aspect ratio for photos has been 3:2 = 1.50:1. Some photographers, like the great Henri Cartier-Bresson, refused to crop their pictures, insisting on the authenticity of the full frame. And the aspect ratio is not even the Golden Mean!

The vocal community convinced today of the importance of the Golden Mean asserts that the advertising industry, realizing the magical attraction of the Golden Mean, manufactures cereal boxes and the *iPhone* to this standard, and there is even a propaganda movie that includes these examples (Spinak, 2011). All not true! The *Apple iPhone 4* has physical dimensions of 115.2 mm x 58.6 mm, thus an aspect ratio of 1.97:1. Its screen, on the other hand, contains 960x640 pixels, giving it an aspect ratio of 1.50:1. As for cereal boxes, they do come in a variety of different shapes and sizes. For the sake of scientific completeness, measurements of a *Weetabix* box (By Appointment To Her Majesty The Queen) give its dimensions at 26.0 cm x 19.0 cm, hence an aspect ratio of 1.37:1.

8. The Parthenon.

The Parthenon in Athens, built by Iktinos and Kallikrates around 440 BC, is widely perceived as an extremely attractive building. I claim that one of several reasons for its appeal is its scaling hierarchy and highly ordered complexity (Salingaros, 1997). And yet Golden Mean enthusiasts attribute its informational success to its supposed design using Golden Mean rectangles, and the Parthenon is promoted as being one of the paradigmatic examples of buildings designed according to the Golden Mean. That claim is founded on several simple misunderstandings.

As is well known, one of the marvels of the Parthenon is its carefully-computed curvature, or “entasis” (Haselberger, 1985). It makes no sense to look for rectangles on a building that is essentially curved. Its frontal façade is not rectangular — it is a distorted rectangle sitting on curved steps, and with a curved triangle on top of it (Figure 5). One cannot define an exact rectangle on the front or back faces of the Parthenon. Even though the Parthenon is built to extremely accurate specifications, its curvature precludes rectangular measurements of any greater precision than about 1%. This built-in error precludes finding any Golden Mean rectangles, since the required accuracy is simply not attainable.



Figure 5. The Parthenon's deliberately curved façade exaggerated to show “entasis”.

Mathematician Keith Devlin dismisses claims that the Golden Mean has anything to do with the Parthenon's design: "*Certainly, the oft repeated assertion that the Parthenon in Athens is based on the golden ratio is not supported by actual measurements. In fact, the entire story about the Greeks and golden ratio seems to be without foundation.*" (Devlin, 2006: p. 108). In two articles, Devlin re-iterates his conclusion that there is not even a shred of evidence to support the appearance of the Golden Mean in art and architecture (Devlin, 2004; 2007). Note that the presence of the Golden Mean in the Parthenon was postulated by Adolf Zeising in the 1850s, and appears nowhere in ancient Greek architectural treatises.

I should not even have to discuss this point since investigators determined a modular measurement system for the Parthenon. A very careful geometrical analysis undertaken by Ernst Berger (Berger, 1980) reveals that the Parthenon's plan is based on a module of 85.76 cm, used in the context of a proportional ratio $9:4 = 2.25:1$. Significantly, Berger did not find any evidence of the Golden Mean. Further analysis by Anne Bulckens independently confirmed this design module, and established the rectangular part of the Parthenon's façade to have an exact $9:4$ aspect ratio before the implementation of curvature to achieve entasis (Bulckens, 2002; Kappraff, 2002).



Figure 6. The obvious rectangle in the Parthenon's façade was most probably built to have exact aspect ratio $9:4$ before curving to achieve "entasis".

Let's look at two different possibilities. First, the obvious rectangle enclosing the columns and entablature (Figure 6) has width $30.88 \text{ m} \pm 135 \text{ mm}$, with the second number representing the total inclination at the top (counting both corners), and height $13.72 \text{ m} \pm 66 \text{ mm}$, which notes the vertical rise through curvature. This gives an aspect ratio of $(2.25 \pm 0.02):1$, where the curvature leads to an "error" of 1%. Therefore, the most obvious rectangle in the Parthenon's façade is unrelated to the Golden Mean, while agreeing completely with the rigorous analysis of Berger and Bulckens.

Next, Markowsky uses measurements to the exterior tip of the triangular pediment (projected, since it no longer exists) to obtain a rectangle with aspect ratio approximately $1.71:1$ (again, a more precise figure is not unique), which is far from the Golden Mean (Markowsky, 1992). It is possible by trial-and-error to superimpose a rectangle that has aspect ratio nearer ϕ , but it is not clear if there is anything to be learned from such an obviously rigged exercise (Figure 7).

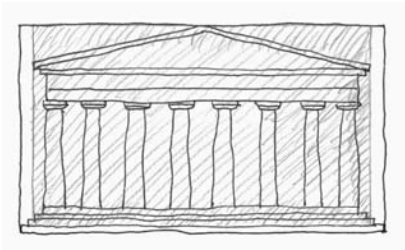


Figure 7. Some Golden Mean enthusiasts draw this shaded rectangle on the Parthenon's façade, using the bottom of the third step. Others only go to the bottom of the first step. Others still, use the bottom of the building's base (a thinner slab that is not a fourth step: there are only three marble steps) and width somewhere out in space.

One sees the now standard published images of the front façade of the Parthenon with a Golden Mean rectangle superimposed — of three different possible sizes and positions! Those rather arbitrary figures are entirely misleading because of the thickness of the line with which such a rectangle is usually drawn. The resulting error is so great as to make any accurate aspect ratio meaningless, and Alexander already pointed out this misconception (Alexander, 1959).

Nature photographer Mike Spinak reaches the same conclusion: *“The notion that the Parthenon matches up with the golden ratio doesn't hold up to examination... The decisions of what to include within the rectangle and what to exclude from it are entirely arbitrary... It's not a close fit; it's just made to look like one by arbitrary inclusion and exclusion... Golden numberists often use thick lines in their demonstrations because they cover up a wide margin of error [of about] 3.33%... The Parthenon does not seem to have golden mean proportions.”* (Spinak, 2011).

9. The Villa Stein/Garches.

The Villa Stein-de-Monzie in Garches-Vaucresson outside Paris, built in 1927 by the Swiss architect Le Corbusier, has a horizontal rectangular façade. It presents a rectangle built with rather well-controlled tolerances. Le Corbusier intended an aspect ratio of 8:5 rather than the Golden Mean ϕ in designing this building, contrary to what he claimed in his voluminous writings on the subject. Available measurements confirm this by giving an aspect ratio that is somewhat less than ϕ ($8/5 = 1.600 < 1.618$).

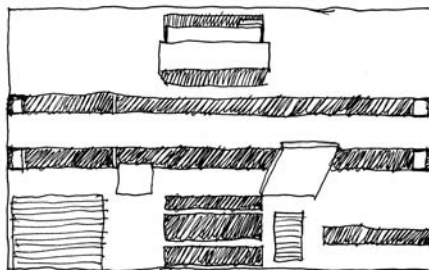


Figure 8. The villa Stein/Garches, in which every component is intentionally designed to be unnatural and not to harmonize.

Roger Herz-Fischler discovered that Le Corbusier went back to the original drawings one and a-half years after the building was completed, and drew in the Golden Mean *ex post facto* (Herz-Fischler, 1984). Le Corbusier subsequently reproduced this doctored drawing, claiming that his design was guided by the Golden Mean all along. Possibly, Le Corbusier was practicing an intentional deception; alternatively, according to Herz-Fischler, Le Corbusier did not distinguish between the 8:5 aspect ratio and the Golden Mean ϕ . Or maybe he just thought that alluding to the Golden Mean would make better publicity than to the more ordinary aspect ratio 8:5.

In the final analysis, does this building's rectangular façade elicit a pleasurable emotional response? I find this building, along with other creations by Le Corbusier, willfully odd and particularly unappealing. It is an example of what Michael Mehaffy and I have labeled as "Geometrical Fundamentalism" (Mehaffy & Salingaros, 2002). To claim any attractive effect from a Golden Mean rectangle (built to any accuracy) is unfounded. What's missing that would make it look appealing and not so bleak, possibly a nice house to live in? Among other things, it lacks a natural scaling hierarchy, adaptation to human purposes, and harmonization with its surroundings. A contemporary house by Le Corbusier, the Villa Savoye, 1929, was declared by its owners to be uninhabitable (Sully, 2009).

10. The United Nations Building.

The geometry of tall buildings cannot be perceived by their users as designed, because of the visual distortion due to perspective (Spinak, 2011). Users inside a building cannot see its form at all. Any building in the form of a vertical rectangular slab is perceived as a trapezoid (a triangle with its top cut off) by anyone nearby, and not as a rectangle (Figure 9). The perceived trapezoid changes as one moves around in the vicinity of the building. Only viewers from sufficiently far away actually see a rectangle. Thus, aesthetic considerations based upon a tall building's overall form apply strictly to viewers at a distance, and then only if they have an unobstructed on-axis view.

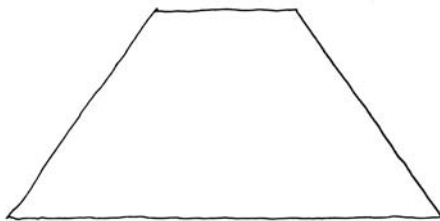


Figure 9. Where's the aspect ratio? Users perceive a tall building built with a rectangular façade as a trapezoid, whose shape changes as the observer's position changes. This building could have any number of storeys.

The Secretariat Building of the United Nations, in New York City, has a façade in the shape of an empty rectangular slab, which is frequently cited as having aspect ratio the Golden Mean. It was designed in 1950 by Wallace Harrison and Max Abramovitz, based on earlier sketches by Le Corbusier. Markowsky computed the aspect ratio of its façade and found that it equals 1.76:1 (Markowsky, 1992). This is not the Golden Mean. Herz-Fischler thinks that Le Corbusier’s original sketches were intended to set up a Golden Mean aspect ratio, but Le Corbusier was not allowed to work on the final design because he got into a nasty fight with Wallace Harrison.

We need to ask two questions, regardless of the validity of any claim based upon the building’s dimensions: 1. Is this gigantic vertical rectangular slab attractive or not?; and 2. Would an adjustment of its height or width to better approximate a Golden Mean aspect ratio make it more attractive than it is at present? The answer to the first question remains in the eye of the beholder, and I personally find this building unbearably dull. Any building as an undifferentiated rectangular slab devoid of hierarchical scaling is just another example of “Geometrical Fundamentalism”, and as such, cannot connect to human beings through complex ordered information (Mehaffy & Salingaros, 2002).

The second question, however, can be confidently answered in the negative: no, any minor adjustment in this building’s proportions to bring it more in line with a Golden Mean rectangle will have absolutely no effect in its actual aesthetic appeal. I conclude that the search for such an aspect ratio has little meaning.

11. Le Corbusier’s Modulor.

Le Corbusier made grandiose claims for a scheme of hierarchical subdivisions that he called *Le Modulor*, which was based on the Golden Mean. No one today supports that it led to any practical designs which are in any way superior to designs that did not use the Modulor. Le Corbusier claimed to have used the Modulor in designing his Unité d’Habitation apartment block in Marseilles, 1947-1952, but people who have measured the dimensions have found discrepancies. The chief flaw of his confused system is that subdivisions are too close to be distinguished psychologically, and so the result is rather chaotic.

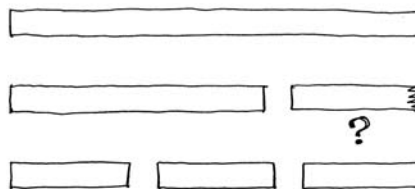


Figure 10. Partition of a system into parts: using the Golden Mean doesn’t work, whereas it’s easy to create three components plus space for their connections.

A scaling hierarchy of sizes cannot be based on the Golden Mean (Salingaros, 2010) (Figure 10). In system decomposition, components are objects of equivalent size that together form the system. The whole can be partitioned into two, three, four or more parts, allowing for the connections that hold them together. A similar decomposition

occurs on each distinct level to define components of components, etc. In a visual/geometrical system such as architecture, a scaling ratio relates structural scales. Possible partitions and their rough scaling ratios are: into two parts with scaling ratio 2.3, four parts with scaling ratio 7 (Figure 2), or three parts with scaling ratio 3.5 (Figure 10). The minimum possible scaling ratio is 2. It's impossible to create subunits at a scaling ratio of 1.618 (Figure 10).

Malcolm Millais, in his forthcoming book on Le Corbusier, is neither kind to the architect, nor to his design system as it was supposedly applied to the Unité d'Habitation. After a careful analysis of the structure, Millais concludes that: *"In fact the dimensions were an incoherent mixture of the two Modulor numerical series, with other dimensions which were odd combinations or not part of the system at all. An examination of the three-storey module cross-sectional dimensions shows what nonsense it all was."* (Millais, 2012).

Le Corbusier copied many of the Unité's key features from the very similar *Narkomfin* apartment building in Moscow, 1928-1932, which he had studied during his visits there in the 1930s. These include the central axis, a flat concrete roof (optimistically labeled "roof garden"), and originally standing the *Narkomfin* building on pilotis (columns) until the ground floor was filled in with more apartments later. Its architects, Moisei Ginzburg and Ignati Milinis, did not use the Modulor in designing the *Narkomfin* block, since that was only published in 1948.

In his brilliant discussion of the Golden Mean in architecture, Alexander had this to say about Le Corbusier and the Modulor:

"Yet we have only to examine the work in detail to see how flimsy its foundations are. The failure of writers to appreciate the true reason for the visual efficacy of the golden section has led them to shelter in a maze of obscurity... First of all, throughout the writings we are concerned with, there seems to be deliberate intention to hoodwink the reader... because the only way to prove things that are incorrect is by false argument. Or perhaps it is simply that the writers are too ignorant to know what they are doing. Le Corbusier, for instance, reverently reproduces facsimiles of two pages of arithmetic a mathematician did for him. The arithmetic involved could have been done by many schoolboys, and to suggest that it is difficult by showing readers the original manuscript is sheer deceit." (Alexander, 1959).

We must be wary of taking anything Le Corbusier wrote at face value because of his record of mendaciousness. The architect and author Anthony Antoniadis interviewed Le Corbusier's apprentices in Paris, who told him that *"Le Corbusier had no idea of mathematics, contrary to what he professed."* (Antoniades, 1990: p. 270). Antoniadis further mentions several negative characteristics of Le Corbusier, such as his plagiarism, pretensions, absence of credibility, and promoting himself by "stepping on cadavers", as revealed by those he worked with (Antoniades, 1990: p. 285). Le Corbusier falsified photographs that he published in his commercial advertizing magazine, later included in his books (Colomina, 1994; Mehaffy & Salingaros, 2002; Millais, 2009). The writer and physician Theodore Dalrymple also condemns Le Corbusier (Dalrymple, 2009a; 2009b).

12. Conclusion.

In this essay, I discussed a method for checking a design to make sure that it possesses a natural hierarchy of scales. Coherent hierarchical structures are found throughout historical and vernacular architectures, but are almost entirely absent from

the formal architecture of the 20th and 21st centuries. A return to more life-enhancing buildings and urban spaces can well use this design method coming from natural growth and complex systems. Scaling in an ordered hierarchy of sizes receives support from the widely-found occurrences of the Golden Mean in natural systems (Livio, 2003). We see growth either as a continuous exponential or discrete Fibonacci sequence very clearly in patterns in the form of plants and animals. There are also natural inanimate systems whose mathematical description leads them to exhibit the Golden Mean.

This application of the Golden Mean to design contrasts with the unsatisfactory state of affairs — or rather, problems in finding any applications at all — when it comes to claims made in the architectural and popular literatures. Rectangles with aspect ratio the Golden Mean have no special meaning; most embarrassing for its numerous proponents is that the key examples used to illustrate this recurring theme turn out to be contrived. Claims for attractive properties of minimalist architecture, which dogmatically celebrates empty rectangles, are therefore nothing more than wishful thinking by its practitioners and their cult followers.

It is frustrating, however, to try and argue for the non-existence of something: far easier to show its existence, since then only a single example suffices. To prove the opposite, an author needs to carefully examine and disprove a number of proposed but erroneous applications. This was attempted here in part, although it would be a fruitless exercise to tackle all the various claims for the appearance of Golden Mean rectangles in art and architecture. More important is to expose the misplaced beliefs that lead to those claims, in the hope of introducing some clarity of thought to the discipline.

Diverging from a study on the applications of mathematics to architecture, this paper instead ends up clarifying certain cult tendencies. By accepting a flawed and rigid model as the accepted “basis for beauty”, the biological basis for genuine beauty is replaced, and the results are unnatural. Alexander described this understanding succinctly:

“We shall examine the cult of the golden section; and show that the claims made for it are in large part exaggerated — that the order this system does provide can be provided just as well by countless other systems which are only less well known because no attempt has been made to mysticize them, to make religions of them... now, at the moment when hope of understanding visual aesthetics is just appearing, the architectural world has been inundated by further mysterious writing on the golden section and geometry. Instead of trying to account for the effect of order in a way appropriate to our time, the majority of writers have returned to an almost primitive acceptance of magic and ritual.” (Alexander, 1959).

Much discussion of the Golden Mean in architecture is plainly wrong, and is spent on analyzing historical buildings to identify the presence of Golden Mean rectangles. I tried to fix misconceptions and errors associated with the Golden Mean in architecture, and turn this into a creative design project. Significantly, architects are finally asking how to build structures today of a comparable emotional appeal. The mathematics of architectural form — the hard work of learning how to build great buildings — combines the latest scientific understandings from complex systems with knowledge revived from traditional design and construction. This new framework provides a useful design basis for generating attractive architectural forms and spaces.

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